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The Explicit Inverses of CUPL-Toeplitz and CUPL-Hankel Matrices

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Abstract. In this paper, we consider two innovative structured matrices, CUPL-Toeplitz matrix and CUPL-Hankel matrix. The inverses of CUPL-Toeplitz and CUPL-Hankel matrices can be expressed by the Gohberg-Heinig type formulas, and the stability of the inverse matrices is verified in terms of 1-, ∞- and 2-norms, respectively. In addition, two algorithms for the inverses of CUPL-Toeplitz and CUPL-Hankel matrices are given and examples are provided to verify the feasibility of these algorithms.

AMS subject classifications: 15A09, 15B05

Key words: CUPL-Toeplitz matrix, CUPL-Hankel matrix, inverse, stability, algorithm.

1. Introduction

As is well known, Toeplitz matrices family are also structured matrices family and have important applications in various disciplines including the elliptic Dirichlet-periodic boundary value problems [1], sinc discretizations of partial and ordinary differential equations [2–7], signal processing [8], numerical analysis [8], system theory [8], etc.

It is an ideal research area and hot issue to find the inverse of a Toeplitz matrix. In [9], Jiang and Wang firstly present an innovative structured matrix, RFPL-Toeplitz matrix, the group inverse of this new structured matrix can be represented as the sum of products of lower and upper triangular Toeplitz matrices, then the explicit expression and the decomposition of the group inverse is given. It turns out the inversion of Toeplitz matrix can be reconstructed by a low number of its columns and the entries of the original Toeplitz matrix. The result was first observed by Trench [10] and reconstructed by Gohberg and Semencul [11] from its first and last columns of T^{-1}, provided that the first component in the first column is not zero. The algorithm of Trench for the inversion of Toeplitz matrices is presented with a detailed proof in [12]. Gohberg and Krupnik [13] observed that

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if the last component of the first column is not zero, then T^{-1} can be recovered from its first and second columns. Heinig and Rost [14] exhibited an inversion formula for every nonsingular Toeplitz matrix, which requires the solution of fundamental equations, where the right-hand side of one of them is a shifted column of the Toeplitz matrix T. In [15], the inverse was reconstructed through three columns of T^{-1}. Labahn and Ng modified this result in [16] and [17]. In [18], the inverse of the Toeplitz matrix was presented in the form of Toeplitz Bezoutian of two columns. Lv and Huang [19] gave a new Toeplitz matrix inversion formula in which the inverse can be denoted as a sum of products of circulant matrices and upper triangular Toeplitz matrices. Labahn [20] proposed formulas for the inverses of layered or striped Toeplitz matrices in terms of solutions of standard equations. The explicit inverses of nonsingular conjugate-Toeplitz and conjugate-Hankel matrices are provided in [21].

In [22] and [23], the stability of the algorithms emerging from Toeplitz matrix inversion formulas was considered. Xie and Wei [24] proposed a stability analysis of Gohberg-Semencul-Trench type formula for Moore-Penrose and group inverses of Toeplitz matrices.

In this paper, we present the explicit inverses of CUPL-Toeplitz and CUPL-Hankel matrices, which can be expressed as sum of products of circulant and upper triangular Toeplitz matrices, which is thought of a Gohberg-Heinig type formula for the inverse of an CUPL-Toeplitz matrix and CUPL-Hankel matrix. Moreover, the stability of the inverse formula is verified in terms of 1-, ∞- and 2-norms, respectively. Then the algorithms of the inverse formulas are provided. And in the final, examples are given to support the feasibility of the algorithms.

Definition 1.1. An $n\times n$ column upper-plus-lower Toeplitz matrix with the first row $(a_0, a_{-1}, a_{-2}, \cdots, a_{1-n})$ and the first column $(a_0, a_1, a_2, \cdots, a_{n-1})^T$ is meant a matrix of the form as

$$T_{CUPL} = \begin{pmatrix} a_0 & a_{-1} & a_{-2} & \cdots & a_{1-n} \\ a_1 & a_0 + a_1 & \ddots & \ddots & \vdots \\ a_2 & a_1 + a_2 & \ddots & \ddots & a_{-2} \\ \vdots & \vdots & \ddots & \ddots & a_{-1} \\ a_{n-1} & a_{n-2} + a_{n-1} & \cdots & a_1 + a_2 & a_0 + a_1 \end{pmatrix}, \tag{1.1}$$

denoted by $T_{CUPL}[fr(a_0, a_{-1}, a_{-2}, \cdots, a_{2-n}, a_{1-n});\ fc(a_0, a_1, a_2, \cdots, a_{n-2}, a_{n-1})^T]$, or by T_{CUPL} for short, where $a_0,\ a_{\pm 1},\ a_{\pm 2}, \cdots, a_{\pm(n-1)}$ are any complex numbers.

Obviously, the entries a_{ij} of the matrix in (1.1) are given by the following formulas:

$$a_{ij} = \begin{cases} a_{i-j}, & j = 1 \text{ or } j > i \\ a_{i-j} + a_{i-j+1}, & 2 \le j \le i. \end{cases} \tag{1.2}$$

Specially, if $a_{1-n} = a_1,\ a_{2-n} = a_2,\ \cdots,\ a_{-1} = a_{n-1}$, then T_{CUPL} is a row first-plus-last right circulant matrix, which is firstly defined in [25].