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Error Control Based on the Novel Proof of Convergence of the MSMAOR Methods for the LCP

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Abstract. The convergence of modulus-based synchronous multisplitting accelerated overrelaxation iteration methods for linear complementarity problems is studied using the new technique by Zhang, Zhang and Ren. We show that this technique is particularly convenient in the a priori and a posteriori error analysis.

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1. Introduction

Linear complementarity problems (LCP) often arise in practical applications, including engineering, numerical optimization, market equilibrium models, and so on. The elevated demand for solving large sparse linear complementarity problems on parallel multiprocessor systems lead to a rapid development of fast and efficient iterative methods. One class of new algorithms based on a suitable fixed-point representation, allows the application of splitting and multisplitting iteration methods to linear systems – cf. [3–5]. In connection to the concept of H_+-matrices, the study of such multisplitting methods for a wide class of large sparse systems containing, in particular, linear nonsymmetric complementarity problems has been started by Bai [1]. The modulus-based splitting iteration method introduced by Bai [2], presents a general systematic approach to modulus iteration [16, 17], modified modulus iteration [12] and nonstationary extrapolated modulus iteration [13, 14]. In particular, the modulus-based synchronous multisplitting iterations, developed by Bai and Zhang [7] and employing the multiple splitting of the system matrices [6, 18], offer synchronous parallel counterparts in such iteration methods. The set of such modulus type algorithms is quite wide and contains known multisplitting relaxation methods such as Jacobi, Gauss-Seidel, SOR and AOR. On the other hand, the convergence of these methods

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has been studied in [7] under the assumption that the system matrix is an H_+-matrix and one acceleration parameter is greater than the other. Cvetković and Kostić [11] pointed out that the latter assumption is too restrictive and can be loosened, thus widening the choice of the relaxation parameters which would guarantee the convergence. A new proof of the convergence, suggested by Zhang *et al.* [21], uses the same presumptions of [7]. In this paper we show that the approaches of [11] and [7] can be combined, which allows to obtain new results on the convergence analysis of the method.

By $\mathbf{N}_n$ we denote the set of the first n natural numbers – i.e. $\mathbf{N}_n := \{1, 2, \cdots, n\}$. If $A = [a_{ij}]$ and $B = [b_{ij}]$ are $m \times n$ matrices, then the notation $A \geq B$ or $A > B$, respectively, means that $a_{ij} \geq b_{ij}$ or $a_{ij} > b_{ij}$ for all $i \in \mathbf{N}_m$ and $j \in \mathbf{N}_n$. Analogously, $|A| := [|a_{ij}|]$. Let $\rho(A)$ refer to the spectral radius of the matrix A.

Lemma 1.1 (cf. Varga [20]). *Let $A \in \mathbb{R}^{n\times n}$ be a non-negative matrix. If there is a positive vector $u \in \mathbb{R}^n$ such that $Au < u$, then $\rho(A) < 1$.*

A real-valued matrix $A \in \mathbb{R}^{n\times n}$ is called a Z-matrix if all off-diagonal entries of A are non-positive. A non-singular Z-matrix is referred to as M-matrix if $A^{-1} \geq O$, where O denotes the zero matrix. The class of M-matrices can be characterised as follows.

Lemma 1.2 (cf. Berman & Plemmons [8]). *A Z-matrix $A \in \mathbb{R}^{n\times n}$ is an M-matrix if and only if there exists a positive vector $u \in \mathbb{R}^n$ such that $Au > 0$.*

By I we denote the identity matrix.

Lemma 1.3 (cf. Berman & Plemmons [8]). *If μ is a positive number and matrix $B \geq O$, then $A = \mu I - B$ is an M-matrix if and only if $\rho(B) < \mu$.*

For any matrix $A \in \mathbb{R}^{n\times n}$, by $\langle A\rangle$ we denote the corresponding comparison matrix – i.e. the matrix $[m_{ij}]$ with the entries $m_{ii} = |a_{ii}|$ and $m_{ij} = -|a_{ij}|$ if $i \neq j$. A non-singular matrix $A = [a_{ij}] \in \mathbb{R}^{n\times n}$ is called an H-matrix if $\langle A\rangle$ is an M-matrix. An H-matrix is called H_+-matrix if all of its diagonal entries are positive. Let us recall that the diagonal entries of any H-matrix A are not equal to zero, $|A^{-1}| \leq \langle A\rangle^{-1}$ and if $A = D - B$ is the splitting of A into diagonal and off-diagonal parts D and B, then $\rho(|D|^{-1}|B|) < 1$.

Let A, M and N be $n \times n$ matrices such that $A = M - N$. We say that the representation $A = M - N$ is the splitting of A if M is a non-singular matrix. Consider now the splittings $A = M_p - N_p$, $p = 1, 2, \cdots, \ell$ of a matrix $A \in \mathbb{R}^{n\times n}$, $l \leq n$ and a partition $\sum_{p=1}^{\ell} E_p = I$ of the identity matrix I by non-negative matrices $E_p \in \mathbb{R}^{n\times n}$. The family of triples (M_p, N_p, E_p), $p = 1, 2, \cdots, \ell$ is termed the multisplitting of A and with the weighting matrices E_p. In addition, the family of triples $(D - L_p, U_p, E_p)$, $p = 1, 2, \cdots, \ell$ such that $A = D - L_p - U_p$ is called the triangular multisplitting of A, if L_p and U_p are, respectively, strictly lower-triangular and zero diagonal matrices for each $p = 1, 2, \cdots, \ell$ and D is the diagonal matrix containing the diagonal entries of A.

The paper is organized as follows. After a brief introduction, we turn our attention to the MSMAOR iteration method. Section 2 considers the formulation of this method as well as the convergence results from [7,21] and shows that the methods of [11] are also suitable