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Numerical Simulation of Two-Layer Shallow Water Flows. Exchange Flow in the Lombok Strait

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Abstract. The momentum-conserving staggered-grid (MCS) scheme for two-layer shallow flow is revisited here. In the previous contribution, the MCS scheme has been successfully used to simulate various steady interfaces of two-layer shallow flow in channels of varying depth and width. In this contribution, the MCS scheme is extended to accommodate a time-dependent barotropic forcing, the barotropic tidal wave is assumed to be a harmonic function. In the MCS scheme, this barotropic forcing is assessed by applying suitable boundary conditions, and its presence would affect the exchange flow. The average exchange transport was then calculated for the barotropic forces with a range of amplitudes and periods, and the results obtained show good agreement with the analytical rigid-lid theory. Next, the MCS scheme is used to study the exchange flow in the Lombok Strait. Taking into account the geographical conditions of the Lombok Strait, a steady interface was simulated, which turned out to be the case of a maximal exchange flow with two controls at Karangasem Narrows (KN) and Nusa Penida Sill (NPS). Next, due to tidal waves in the period of April-May, the average exchange flow in Lombok Strait is estimated. Our findings agree well with the existing results.

AMS subject classifications: 76B55, 37N10, 86-10
Key words: Two-layer shallow-water equation, maximal exchange flow, time-dependent barotropic flow, momentum-conserving staggered grid scheme.

1. Introduction

The Lombok Strait is a narrow passage that separates the islands of Bali and Lombok in Indonesia. It is an important shipping waterway that connects the Flores Sea to the

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north and the Indian Ocean to the south. The strait is approximately 40 kilometers at its widest point and has a special feature of a sill with a depth of around 250 m. The currents in the Lombok Strait are complex and highly variable, with a number of different factors contributing to their behavior, such as wind, tides, and oceanographic conditions. One of the main drivers of the currents in the strait is the Indonesian Throughflow, which is a current system that transports warm water from the Pacific Ocean to the Indian Ocean through a series of passages and straits in the Indonesian archipelago [13]. In general, currents in the Lombok Strait tend to flow southward from the Flores Sea to the Indian Ocean. However, this current may reverse thus can be observed, one of them, as a pack of internal waves propagating northward, see Fig. 1. During change of monsoon April-May and November, Kelvin Waves are generated in the tropical Indian Ocean and propagate eastward along the west coast of Sumatra until the south coast of lesser Sunda islands [23,24]. When passing Lombok strait, coastally trapped Kelvin Waves generate a counter-current northward and can be observed up to 150 m with a delay of 2-3 weeks after the generation [12,18,22,23]. Therefore, it is important to note that the currents in the Lombok Strait can be highly variable and are influenced by a number of different factors, so there may be times when the direction of the currents deviates from this general pattern.

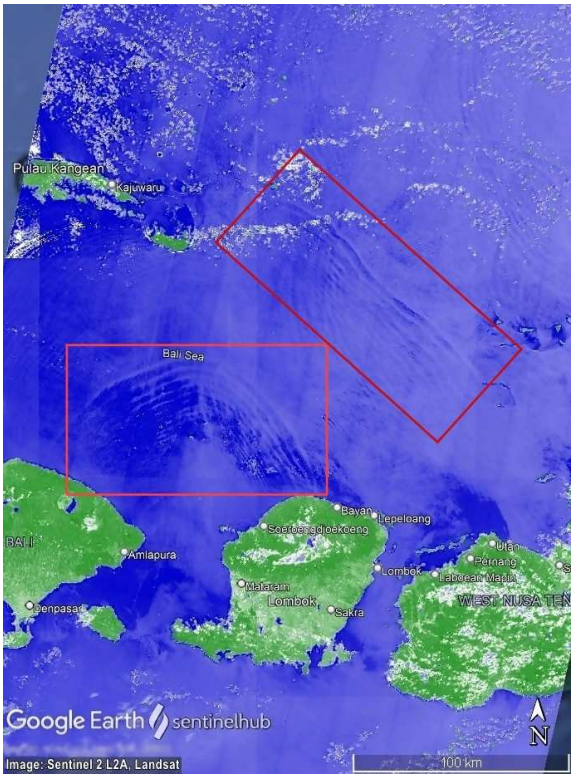

Figure 1: Internal waves propagating away from Lombok strait captured by Sentinel 2 on 17 April 2021 (modified in Copernicus Sentinel data 2023/Sentinel Hub).