NUMERICAL COMPUTATION OF THE FLOW WITH A SHOCK WAVE PASSING THROUGH A "STRONG EXPLOSION" CENTER*

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1. Introduction

When a strong plane explosion wave impinges normally upon a rigid wall, a plane reflected wave forms. It propagates into the region disturbed by the explosion. This paper is a sequel to [2] and [4]. We use exact quasilinear equations of gasdynamics with Euler's coordinate and time and exact gasdynamic shock conditions. The problem was treated by many existing excellent methods. But we realized that without special treatment it is hardly possible to compute the propagation of the reflected wave until it reaches the explosion center, and it is even more difficult to calculate further propagation of the reflected shock. The difficulty is due to the fact that in the model of the so-called "point explosion", the gas temperature and the speed of propagation of the shock wave tend to infinity as the explosion center is approached. On the basis of analysis of the singular behavior of the solution and further development of the "singularity-separating method" we successfully overcome the above mentioned difficulty. In this paper the results of the flow behind the reflected shock are presented, describing how the reflected wave reaches and passes the explosion center. Further results of the flow field are obtained for all time until the reflected wave propagates to a distance about three times the distance from the wall to the explosion center. After the reflected wave interacts with the explosion center, one should treat carefully a curved trajectory with vanishing density as well as a new shock wave propagating towards the rigid wall. The new shock forms from the intersection of the characteristic lines of the same second family. Numerical results are satisfactory, which means that our singularity-separating method is efficient for solving this kind of problems with rather complex singularity.

2. Formulation of the Problem

Huang has presented the formulation of our problem of unsteady plane motion of a perfect gas in [2]. In that formulation the following system of equations of gasdynamics is used:

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$$\begin{cases} \dfrac{\partial u}{\partial t}+u\dfrac{\partial u}{\partial x}+\dfrac{1}{\rho}\dfrac{\partial p}{\partial x}=0, \\ \dfrac{\partial \rho}{\partial t}+u\dfrac{\partial \rho}{\partial x}+\rho\dfrac{\partial u}{\partial x}=0, \\ \dfrac{\partial p}{\partial t}+u\dfrac{\partial p}{\partial x}+\gamma p\dfrac{\partial u}{\partial x}=0. \end{cases} \tag{1}$$

Suppose that at time $t=-t_1(t_1>0)$, a "strong" explosion at the plane $x=R_0$ appears. "Strong" explosion means that the initial energy and the pressure of the static gas in front of the explosion wave can be neglected when compared with the energy released per unit area E. The explosion wave AO (Fig. 1) moves towards the rigid wall $x=0$. At $t=0$ it meets the wall $x=0$, and ODB denotes the reflected wave. Thus there are three regions as shown in Fig. 1 on the x-t plane: region I (the region behind the strong plane explosion wave where there is a selfsimilar solution), region II (the unsteady gas flow behind the reflected wave which is neither isentropic nor selfsimilar) and region III ($\rho_0\neq 0,\ p=u=e=0$).

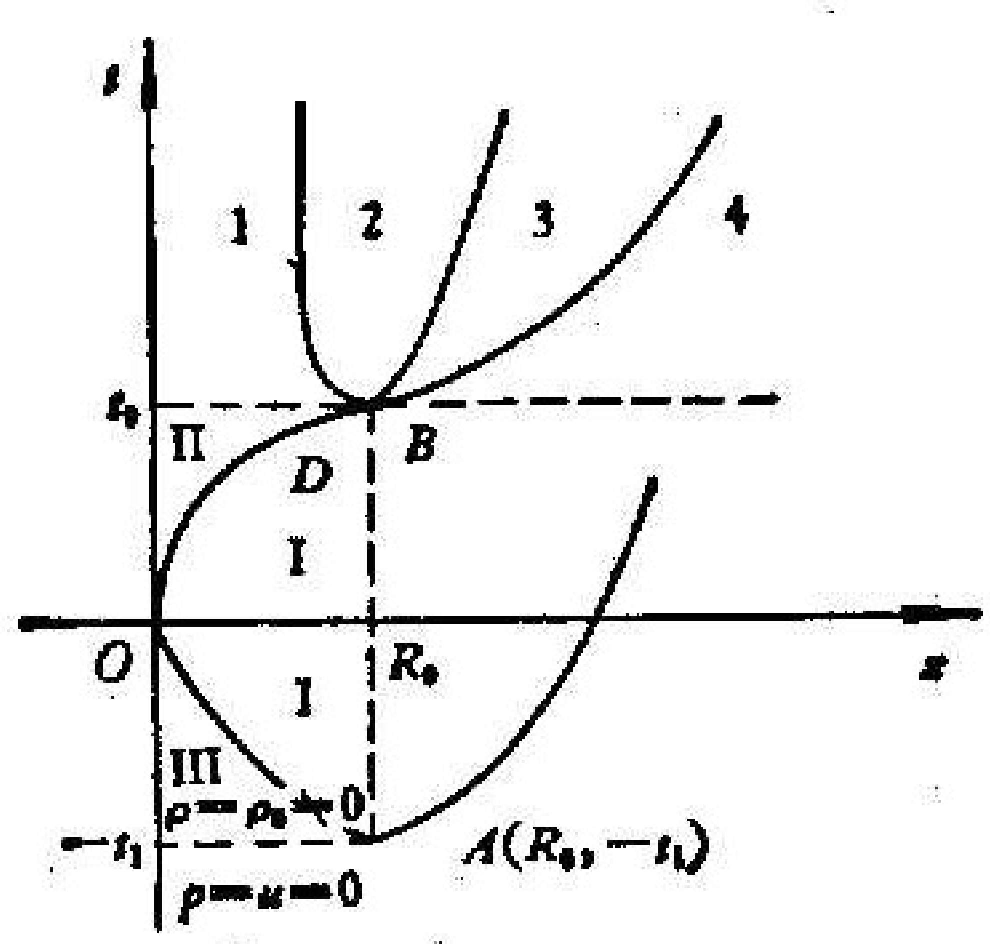

Fig. 1

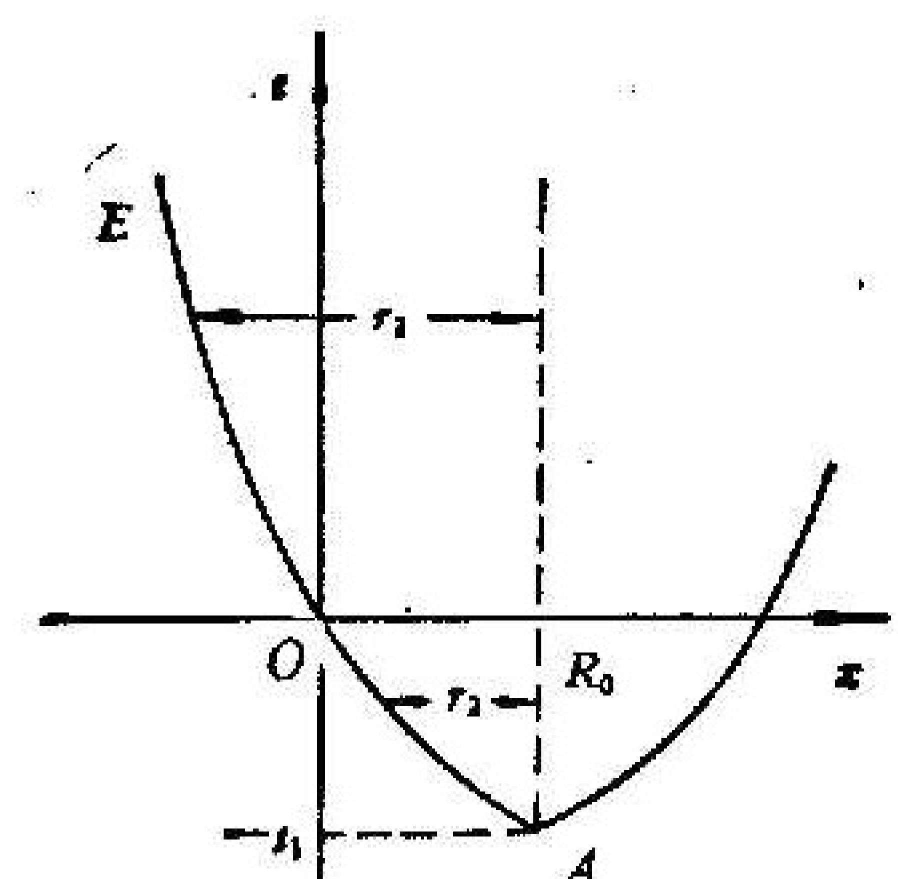

Fig. 2

The selfsimilar solution for region I is given in [2] and [3]: If there is no wall, the wave AO will propagate towards the left as shown by curve AOE in Fig. 2. According to [2], the distance r_2 between AOE and $x=R_0$ is given by

$$r_2=\left(\frac{E}{\rho_0}\right)^{1/3}(t+t_1)^{2/3},$$

then $R_0=\left(\dfrac{E}{\rho_0}\right)^{1/3}t_1^{2/3}$ and

$$\frac{dr_2}{dt}=V=\frac{2}{3}\,\frac{r_2}{t+t_1},$$

V being the speed of the explosion wave. Set $R_0=0.1$ m, $E=2734905.6\ \mathrm{J/m^2}$, $t_1=0.21718193\times 10^{-4}$s, $\rho_0=1.29\ \mathrm{kg/m^3}$ and the specific-heat ratio $\gamma=1.4$. The pressure behind the reflected shock at the origin of x-t diagram is just 800 atm. Since $p=u=e=0$ in region III, immediately behind the incident wave AOE the gas density $\bar{\rho}=6\rho_0$, the particle velocity $\bar{v}=V/1.2$ and the pressure $\bar{p}=\dfrac{1}{1.2}\rho_0 V^2$. Denote $(R_0-x)/r_2$ by $R(x,\ t)$. It relates to a parameter $\bar{V}$ by the relation[3]

$$R=(1.8\bar{V})^{-2/3}(12.6\bar{V}-6)^{2/9}(3-3.6\bar{V})^{-5/9}, \tag{2}$$

then $u,\ p,\ \rho$ in the region I can be expressed by