A TWO-SIDED INTERVAL ITERATIVE METHOD FOR THE FINITE DIMENSIONAL NONLINEAR SYSTEMS*

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Abstract

For the nonlinear system

$$x=g(x)+h(x)+c, \quad x\in R^n,$$

where g and h are isotone and antitone mappings respectively, a two-sided iterative method and the existence theorem of a solution for the system have been given in [2]. In this paper, a two-sided interval iterative method is presented, the initial condition of the two-sided iterative method is relaxed, and the convergence of the two methods are proved.

1.

Consider a nonlinear system

$$x=f(x), \quad x\in R^n, \tag{1.1}$$

where f: $R^n\to R^n$ can be expressed as

$$f(x)=g(x)+h(x)+c, \tag{1.2}$$

where g and h are isotone and antitone mappings respectively, that is, from $x\leqslant y$, we have

$$g(x)\leqslant g(y), \quad h(x)\geqslant h(y).$$

By the two-sided iterative method

$$y^{(k+1)}=g(y^{(k)})+h(z^{(k)})+c,$$
$$z^{(k+1)}=g(z^{(k)})+h(y^{(k)})+c, \quad k=0, 1, \cdots \tag{1.3}$$

the existence of a solution to (1.1) is given in [1] and [2].

Assume that

$$y^{(0)}\leqslant y^{(1)}, \quad z^{(1)}\leqslant z^{(0)}. \tag{1.4}$$

Then there exist points y^*, z^*, such that $y^{(k)}\uparrow y^*$ and $z^{(k)}\downarrow z^*$ as $k\to\infty$. Moreover, any fixed point of the operator $f(x)$ in $[y^{(0)}, z^{(0)}]$ is contained in $[y^*, z^*]$. If $f(x)$ is continuous on $[y^{(0)}, z^{(0)}]$, then there exists a solution of (1.1) in $[y^*, z^*]$.

In general, y^* and z^* are not the solution of (1.1).

A method for finding the initial approximation satisfying (1.4) has been given in [3], which is the key to using the two-sided iterative method.

In order to relax the initial condition of the two-sided iterative method the authors give a two-sided interval iterative method. The initial condition of this method is

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$$[y^{(0)}, z^{(0)}] \not\subseteq [y^{(1)}, z^{(1)}]. \tag{1.5}$$

Clearly condition (1.5) is much weaker than (1.4). Moreover, when (1.4) holds, the two methods will coincide.

In this paper, the convergence of $[y^{(k)}, z^{(k)}]$ to the unique solution of (1.1) is given under condition (1.4). The existence and uniqueness of a solution of (1.1) and the convergence of the two-sided interval iterative method are proved under the condition

$$[y_i^{(0)}, z_i^{(0)}] \not\subset [y_i^{(1)}, z_i^{(1)}], \quad i=1, 2, \cdots, n.$$

Finally, we give a simple example for the two-sided interval iterative method. Under the initial condition which the two-sided iterative method fails to meet, we obtain the existence and uniqueness of a solution of the example after one step of iteration and the approximate solution after 15 steps, with accuracy 10^{-3}.

The notation is as follows. Let $y, z, \bar{y}, \bar{z}, x \in R^n$, $y \leqslant z$, $\bar{y} \leqslant \bar{z}$. Then

$$[y, z] = \{x \mid y \leqslant x \leqslant z\},$$

$$W[y, z] = z - y,$$

$$m[y, z] = 1/2\ (z+y),$$

$$|x| = (|x_1|, |x_2|, \cdots, |x_n|),$$

$$I = \{1, 2, \cdots, n\},$$

$$[\bar{y}, \bar{z}] \subseteq [y, z] \Leftrightarrow y_i \leqslant \bar{y}_i,\ \bar{z}_i \leqslant z_i,\ i=1, 2, \cdots, n,$$

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and there is $i \in I$ such that $\bar{y}_i - y_i + z_i - \bar{z}_i > 0$,

$$[\bar{y}, \bar{z}] \subset [y, z] \Leftrightarrow y_i \leqslant \bar{y}_i,\ \bar{z}_i \leqslant z_i, \text{ and } \bar{y}_i - y_i + z_i - \bar{z}_i > 0,\ i=1, 2, \cdots, n.$$

2.

For $f(x)$ we consider an interval operator

$$F[y, z] = G[y, z] + H[y, z] + c, \tag{2.1}$$

$$G[y, z] = [g(y), g(z)], \quad H[y, z] = [h(z), h(y)].$$

Property 1. F is an inclusion monotonic interval extension of f [4].

Property 2. If $f(x)$ has a fixed point $x^* \in [y, z]$, then $x^* \in F[y, z]$.

Property 3. If $[y, z] \cap F[y, z] = \varnothing$, then there is no solution of (1.1) in $[y, z]$.

Property 4. Suppose f is continuous on $[y, z]$. Then there is a solution of (1.1) in $[y, z]$ as $F[y, z] \subseteq [y, z]$.

By these important properties, we can introduce the two-sided interval iterative algorithm.

Initial step

Define the initial interval $[y^{(0)}, z^{(0)}]$.

1. If $[y^{(0)}, z^{(0)}] \cap F[y^{(0)}, z^{(0)}] = \varnothing$, then the algorithm is stopped.
2. If $[y^{(0)}, z^{(0)}] \cap F[y^{(0)}, z^{(0)}] \neq \varnothing$, then define $[y^{(1)}, z^{(1)}] = [y^{(0)}, z^{(0)}] \cap F[y^{(0)}, z^{(0)}]$.

Continuation step