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FINITE ELEMENT SIMULATION OF INCOMPRESSIBLE FLOW WITH MOVING BOUNDARIES

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Introduction

Flows involving a moving fluid interface are often encountered in many scientific and engineering applications. Extrusion of liquids, problems in capillarity, crystal growth, electrochemical plating and corrosion, metal and glass forming processes, and the coating of solid substrates with liquids are examples of such flows. If the interface is between a gas and a liquid, it is frequently referred to as a free surface.

Computer aided analysis can play a significant role in understanding, operating and controlling such processes in both laboratory and industrial settings. It would enable the analyst/engineer to perform simulations and parametric studies to determine the system configuration and characteristics subject to various different flow rates, temperature profiles, etc. on the computer rather than in the laboratory, and in many instances greatly improve and shorten the design process. To carry out such analyses requires a technique which can accurately represent system variables on a deforming, irregular domain whose free surface portions may be unknown a priori. The Galerkin-finite element technique has aspects which suit it well to such irregular and possible time dependent domain problems since, for example, it can easily accommodate the higher resolution required in certain regions, the singularities associated with contact lines and the complicated boundary conditions associated with such system. The basic approach to problems involving a free surface presented herein involves a deforming spatial mesh where nodes located on a free surface are allowed to move such that they remain on the free surface. An additional degree of freedom is associated with the nodes on the free surface which directly determines their location in space. This is then coupled with a Newton-type iterative procedure which results in the simultaneous calculation of the position of the nodes on the free surface and the field variables at the new nodal positions once convergence is attained.

2. Formulation of the Continuum Problem

The equations of incompressible fluid flow are derived from the basic physical principles of conservation of mass and linear momentum and are the well-known Navier-Stokes equations. The equations for a free surface flow problem are identical except that the location of the free surface is unknown and is determined by additional boundary conditions at the free surface. In the numerical procedure the nodes on the free surface are allowed to move. This requires the addition of a new degree of freedom to the problem to account for the changing shape of the fluid mass.

At a free surface, continuity of stress and velocity is required which leads to the conditions,

$$u_n = 0, \quad x \in \Gamma_f, \tag{1a}$$

$$f_n - p_0 = 2\sigma H, \quad x \in \Gamma_f, \tag{1b}$$

$$f_t = 0, \quad x \in \Gamma_f, \tag{1c}$$

where $u_n = u_i n_i$ is the normal component of velocity, $f_n = \sigma_{ij} n_j n_i$ the normal, and $f_t = \sigma_{ij} n_j t_i$ the tangential component of the stress vector at the boundary, σ is the surface tension, p_0 the pressure in the adjacent vaporphase and H the mean Gaussian curvature of the surface. These conditions can also be written in dimensionless form as:

$$\begin{aligned} &u_n = 0, \\ &f_n = p_0 + \frac{1}{\mathrm{Re}\cdot\mathrm{Ca}} 2H, \\ &f_t = 0, \end{aligned} \tag{2}$$

where u_i, p, p_0 are dimensionless quantities and Ca$= \mu U/\sigma$ is the capillary number.

In the general three-dimensional case, the mean Gaussian curvature of a surface H is equal to $\frac{1}{2}(K_1 + K_2)$, where K_1 and K_2 are the principal curvatures of a surface. It is not necessary to choose the principal curvatures, however it is often more convenient to do so. In the two-dimensional case, the curvature of a "surface" reduces to the curvature of a line with one of the principal curvatures, K_2, being zero. For a line, the curvature vector is,

$$k = Kn = (\nabla \cdot n)n = -\frac{dt}{ds}$$

where n is the normal vector to the line and t is the tangent vector, i.e. the curvature is equal to the change in the tangent vector as one moves along the line.

3. Finite Element Formulation

Since the details of application of the FEM to the basic equations of fluid flow can be found elsewhere [1], this section will concentrate only on those aspects of the procedure introduced as a result of the free surface. At each node on the free surface a new degree of freedom is introduced; the value of this degree of freedom will enable the determination of the position of the node within the region and is