Journal of Computational Mathematics, Vol.9, No.4, 1991, 314–320.

THE SYMMETRICAL DISSIPATIVE DIFFERENCE SCHEMES FOR QUASI-LINEAR HYPERBOLIC CONSERVATION LAWS*

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Abstract

In this paper we construct a new type os symmetrical dissipative difference scheme. Except discontinuity these schemes have uniformly second-order accuracy. For calculation using these, the simple-wave is very exact, the shock has high resolution, the programing is simple and the CPU time is economical.

Since the paper [1] introduced that in some conditions Lax-Wendroff scheme would convergent to nonphysical solution, many researchers have discussed this problem. According to preserve the monotonicity of the solution preserving monotonical schemes and TVD schemes have been introduced by Harten, et. According to property of hyperbolic wave propagation the schemes of split-coefficient matrix(SCM) and split-flux have been formed. We emphasize the dissipative property of scheme for conservation laws and introduced a type of symmetrical dissipative difference scheme, these schemes are dissipative on arbitrary conditions.

§1. The Symmetrical Dissipative Schemes for Hyperbolic Conservation Laws

The quasi-linear conservation law is represented by the following equation

$$\frac{\partial u}{\partial t}+\frac{\partial f}{\partial x}=0 \tag{1}$$

where u, $f(u)$ are column vectors with m dimensions. $A=f_u$ is coefficient matrix of equation (1), it has m real eigenvalues

$$\lambda_1 \le \lambda_2 \le \cdots \le \lambda_m$$

and a complete set of left (right) eigenvectors for all real λ_i.

Advantages of symmetrical scheme is that formula and programming become simpler, that the computation cost is law. We shall see that schemes constructed from the point of view of dissipative property has less severe limitations and often has weaker restriction of stability condition than that from the point of presevering monotonicity of the solution.

In papers [4] and [3], we have constructed the first-order, second-order least dissipative hybrid schemes of presevering monotonicity of the solution

$$u_j^{n+1}=L_2u_j^n+\frac{1}{2}\left[q^n_{j+\frac12}\theta^n_{j+\frac12}(u^n_{j+1}-u^n_j)-q^n_{j-\frac12}\theta^n_{j-\frac12}(u^n_j-u^n_{j-1})\right] \tag{2}$$

* Received July 23, 1988.

where $L_2 u_j^n$ stands for MacCormack two-step scheme or Lax-Wendroff scheme, we have defined

$$\theta_j = \begin{cases} \dfrac{||\Delta\sigma_j| - |\Delta\sigma_{j-1}||}{|\Delta\sigma_j| + |\Delta\sigma_{j-1}|} & \text{for } |\Delta\sigma_j| + |\Delta\sigma_{j-1}| > \varepsilon_0, \\ 0 & \text{for } |\Delta\sigma_j| + |\Delta\sigma_{j-1}| \le \varepsilon_0 \end{cases} \tag{3}$$

and

$$\theta_{j+\frac{1}{2}} = \max\{\theta_j, \theta_{j+1}, \Delta t\}$$

for the single conservation law $\sigma = u$, for Euler equations, $\sigma = \rho$ or c. Assume that

$$\nu_j = \max_{1<k<m} |\lambda_{kj}| \tag{4}$$

we define

$$q = \nu\frac{\Delta t}{\Delta x}\left(1 - \nu\frac{\Delta t}{\Delta x}\right) \tag{5}$$

and

$$q_{j+\frac{1}{2}} = \frac{1}{2}(q_j + q_{j+1}) \tag{6}$$

or

$$q_{j+\frac{1}{2}} = \nu_{j+\frac{1}{2}}\frac{\Delta t}{\Delta x}\left(1 - \nu_{j+\frac{1}{2}}\frac{\Delta t}{\Delta x}\right) \tag{7}$$

where $\nu_{j+\frac{1}{2}}$ is one of the averages of ν_j and ν_{j+1}.

Next we shall construct some new symmetrical dissipative schemes according to the view of dissipative property.

Scheme 1. MacCormack two-step scheme with $\theta/8$ dissipative modification

$$\begin{aligned} &u_j^{\overline{n+1}} = L_2 u_j^n, \\ &u_j^{\overline{n+1}} = u_j^{\overline{n+1}} + \frac{1}{8}\left[\tilde{\theta}_{j+1/2}^{n+1}(u_{j+1}^{\overline{n+1}} - u_j^{\overline{n+1}} - \tilde{\theta}_{j-1/2}^{n+1}(u_j^{\overline{n+1}} - u_{j-1}^{\overline{n+1}})\right] \end{aligned} \tag{8}$$

where

$$\tilde{\theta}_{j+1/2}^{n+1} = \max\{\theta_{j+1}^{\overline{n+1}}, \theta_j^{\overline{n+1}}, 8\eta\Delta t\}.$$

Scheme 2. MacCormack two-step scheme with $\frac{1}{2}\theta q$ dissipative modification

$$\begin{aligned} &u_j^{\overline{n+1}} = L_2 u_j^n, \\ &u_j^{n+1} = u_j^{\overline{n+1}} + \frac{1}{2}\left[q_{j+1/2}^{\overline{n+1}}\theta_{j+1/2}^{\overline{n+1}}(u_{j+1}^{\overline{n+1}} - u_j^{\overline{n+1}}) - q_{j-1/2}^{\overline{n+1}}\theta_{j-1/2}^{\overline{n+1}}(u_j^{\overline{n+1}} - u_{j-1}^{\overline{n+1}})\right]. \end{aligned} \tag{9}$$

Scheme 3. Diffusive-antidiffusive second-order scheme with the least dissipative.

$$\begin{aligned} &u_j^{\overline{n+1}} = L_2 u_j^n + \frac{1}{2}\left[q_{j+1/2}^n(u_{j+1}^n - u_j^n) - q_{j-1/2}^n(u_j^n - u_{j-1}^n)\right], \\ &u_j^{n+1} = u_j^{\overline{n+1}} - \frac{1}{2}\left[q_{j+1/2}^{\overline{n+1}}(u_{j+1}^{\overline{n+1}} - u_j^{\overline{n+1}}) - q_{j-1/2}^{\overline{n+1}}(u_j^{\overline{n+1}} - u_{j-1}^{\overline{n+1}})\right]. \end{aligned} \tag{10}$$

Scheme 4. Modified diffusive-antidiffusive scheme with the least diffispative

$$\begin{aligned} &u_j^{\overline{n+1}} = L_2 u_j^n + \frac{1}{2}\left[q_{j+1/2}^n(u_{j+1}^n - u_j^n) - q_{j-1/2}^n(u_j^n - u_{j-1}^n)\right], \\ &u_j^{n+1} = u_j^{\overline{n+1}} - \frac{1}{2}\left[q_{j+1/2}^{*\overline{n+1}}(u_{j+1}^{\overline{n+1}} - u_j^{\overline{n+1}}) - q_{j-1/2}^{*\overline{n+1}}(u_j^{\overline{n+1}} - u_{j-1}^{\overline{n+1}})\right] \end{aligned} \tag{11}$$