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A HIGHLY ACCURATE NUMERICAL METHOD FOR FLOW PROBLEMS WITH INTERACTIONS OF DISCONTINUITIES*1)

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Abstract

A type of shock fitting method is used to solve some two and three dimensional flow problems with interactions of various discontinuities. The numerical results show that high accuracy is achieved for the flow field, especially at the discontinuities. Comparisons with the Lax-Friedrichs scheme and the ENO scheme confirm the accuracy of the method.

Key words: Interaction of discontinuity, Shock-fitting, Shock-capturing.

1. Introduction

Compared with the widely used shock capturing methods for the compressible flows, the shock fitting methods have the main advantage of accuracy. The shock fitting methods are usually very accurate wherever it can be applied. Several kinds of shock fitting methods have been developed in the last three decades. Glimm and his coworkers have worked extensively on the front tracking methods and applied them to many complicated problems with shocks and other types of singularities [1]-[6]. Moretti [7] has also obtained some very accurate results for gas dynamics problems using his shock fitting method. Mao [8] also developed a front tracking technique for two dimensional problems. In [9]-[10] we developed a technique called the singularity separating method by which the interactions of discontinuities in three dimensional steady supersonic flow has been accurately computed. In [13] we extended the singularity separating method to unsteady two and three dimensional shock reflection problems and obtained very accurate numerical results. The main idea of the singularity separating method is that the computational domain is transformed into sub-domains in which the discontinuities are fixed boundaries. In return, the transformations are time dependent. The flow fields on the two sides of the discontinuities are related by the shock jump conditions or other relations the discontinuities should satisfy. The equations are discretized in each of the sub-domains and no differentiations are performed across the discontinuities. In this way, the high accuracy is achieved both away and at the discontinuities.

In this paper, we extend the method developed in [13] to more complicated problems with interactions of various discontinuities. In section 2, we show the treatment of the interactions of discontinuities. In section 3, we give the algorithms. In section 4, we show two examples of two

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and three dimensional problems with interactions of discontinuities and compare the numerical results with the results obtained from Lax-Friedrichs and ENO schemes.

2. Interactions of Discontinuities

We consider an inviscid flow with some discontinuities in a three dimensional rectangular channel. For simplicity, we assume that at the beginning there are only two discontinuities in the channel. As the time increases, the two discontinuities will get closer, and interact. The governing equation is the three dimensional Euler equation:

$$\frac{\partial \tilde{U}}{\partial t} + \tilde{P}\frac{\partial \tilde{U}}{\partial x} + \tilde{Q}\frac{\partial \tilde{U}}{\partial y} + \tilde{R}\frac{\partial \tilde{U}}{\partial z} = 0, \tag{1}$$

where $\tilde{U} = (p, u, v, w, \rho)^T$,

$$\tilde{P} = \begin{pmatrix} u & \rho a^2 & 0 & 0 & 0 \\ 1/\rho & u & 0 & 0 & 0 \\ 0 & 0 & u & 0 & 0 \\ 0 & 0 & 0 & u & 0 \\ 0 & \rho & 0 & 0 & u \end{pmatrix}, \tilde{Q} = \begin{pmatrix} v & 0 & \rho a^2 & 0 & 0 \\ 0 & v & 0 & 0 & 0 \\ 1/\rho & 0 & v & 0 & 0 \\ 0 & 0 & 0 & v & 0 \\ 0 & 0 & \rho & 0 & v \end{pmatrix}, \tilde{R} = \begin{pmatrix} w & 0 & 0 & \rho a^2 & 0 \\ 0 & w & 0 & 0 & 0 \\ 0 & 0 & w & 0 & 0 \\ 1/\rho & 0 & 0 & w & 0 \\ 0 & 0 & 0 & \rho & w \end{pmatrix},$$

a^2 denoting $\gamma p/\rho$ (in our computation $\gamma = 1.4$).

The initial conditions at $t = 0$ are given by

$$\begin{aligned} & p(x,y,z,0) = p_n(x,y,z), \quad u(x,y,z,0) = u_n(x,y,z), \quad v(x,y,z,0) = v_n(x,y,z), \\ & w(x,y,z,0) = w_n(x,y,z), \quad \rho(x,y,z,0) = \rho_n(x,y,z), \\ & \text{for } \tilde{g}_{n-1}(y,z) \le x \le \tilde{g}_n(y,z),\ -y_b \le y \le y_b,\ -z_b \le z \le z_b,\ n = 1,2,3, \end{aligned}$$

where $p_n(x,y,z)$, $u_n(x,y,z)$, $v_n(x,y,z)$, $w_n(x,y,z)$ and $\rho_n(x,y,z)$, $n = 1,2,3$, are given functions, $\tilde{g}_0(y,z) = 0$ is the left boundary, $\tilde{g}_1(y,z)$ and $\tilde{g}_2(y,z)$ are the two discontinuities, and $\tilde{g}_3(y,z) = 1$ is the right boundary. The flow fields on the two sides of a shock must satisfy the jump conditions:

$$\rho_0(V_{0n} - s) = \rho_1(V_{1n} - s), \tag{2}$$

$$p_0 + \rho_0(V_{0n} - s)^2 = p_1 + \rho_1(V_{1n} - s)^2, \tag{3}$$

$$V_{0t1} = V_{1t1}, \qquad V_{0t2} = V_{1t2}, \tag{4}$$

$$\frac{1}{2}(V_{0n} - s)^2 + \frac{\gamma p_0}{(\gamma - 1)\rho_0} = \frac{1}{2}(V_{1n} - s)^2 + \frac{\gamma p_1}{(\gamma - 1)\rho_1}, \tag{5}$$

where s is the shock speed, V_{0n} and V_{1n} denote the velocities normal to the shock on the two sides of the shock, and V_{0t1}, V_{0t2}, V_{1t1}, and V_{1t2} denote the two linearly independent components