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ANOMALOUS DIFFUSION IN FINITE LENGTH FINGERS COMB FRAME WITH THE EFFECTS OF TIME AND SPACE RIESZ FRACTIONAL CATTANEO-CHRISTOV FLUX AND POISEUILLE FLOW*

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Abstract

This paper presents an investigation on the anomalous diffusion in finite length fingers comb frame, the time and space Riesz fractional Cattaneo-Christov flux is introduced with the Oldroyds' upper convective derivative and the effect of Poiseuille flow is also taken into account. Formulated governing equation possesses the coexisting characteristics of parabolicity and hyperbolicity. Numerical solution is obtained by the L1-scheme and shifted Grünwald formulae, which is verified by introducing a source item to construct an exact solution. The effects, such as time and space fractional parameters, relaxation parameter and the ratio of the pressure gradient and viscosity coefficient, on the spatial and temporal evolution of particles distribution and dynamic characteristics are shown graphically and analyzed in detail.

Mathematics subject classification: 60J60, 97M10, 35R11, 92E20, 97K60.
Key words: Anomalous diffusion, Cattaneo-Christov flux, Fractional derivative, Poiseuille flow, Distribution.

1. Introduction

In this paper, we present a comprehensive research for anomalous diffusion in finite length fingers comb frame [1, 2] (as shown in Fig. 1.1) subject to the time and space Riesz fractional Cattaneo-Christov flux [3,4] and the effects of Poiseuille flow [5] is also taken into account. The diffusion in comb model is a special case of random walk and it has been used to study the

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diffusion of cancer cells [6], the transport along the spiny dendrites [7], the diffusion and drift in percolation clusters [8] and so on. On the basis of previous study, we firstly introduce the convection velocity which is deduced by Poiseuille flow and the time and space Riesz Cattaneo-Christov constitutive relation to analyze and discuss the anomalous diffusion in comb frame.

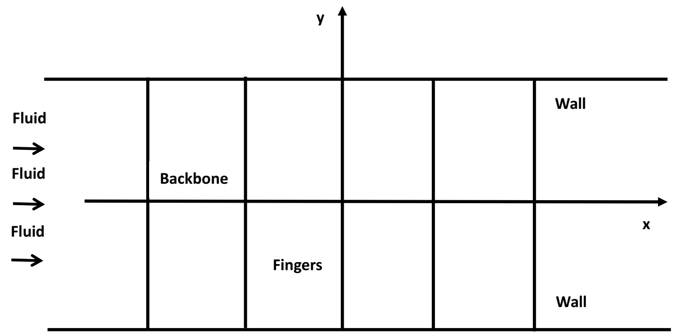

Fig. 1.1. The comb model with finite lengths of fingers.

The previous papers mainly discuss the diffusion in the comb model with the constant velocity or the exponential form [9]. Actually, the convection velocity is a complex process which can be described by the Navier-Stokes equations. One of the simplest forms is called the Poiseuille flow that the flow of incompressible viscous fluid is driven by pressure gradient, and the simple expressions for the velocity distribution are given as follows:

$$u\left(y,t\right)=\frac{K}{2\mu}\left(y^2-h^2\right),\ v=0 \tag{1.1}$$

where $u\left(y,t\right)$ is along the x direction while v is along the y direction, the parameters K, μ and h refer to the pressure gradient, the viscosity coefficient of relevant fluid and the half of the distance between the two walls, respectively. The dimension for K and μ are respectively given as N/m^3 and $N*s/m^2$.

On the basis of the Fick's law [10] and Cattaneo model [11], a newly introduced Cattaneo-Christov constitutive relation is proposed:

$$\mathbf{j}+\xi\left[\frac{\partial\mathbf{j}}{\partial t}+\mathbf{V}\cdot\nabla\mathbf{j}-\mathbf{j}\cdot\nabla\mathbf{V}+\left(\nabla\cdot\mathbf{V}\right)\mathbf{j}\right]=\left\{-D_x\frac{\partial P}{\partial x},-D_y\frac{\partial P}{\partial y}\right\}, \tag{1.2}$$

where $\mathbf{j}$ refers to the diffusion flux, ξ is a nonnegative constant and refers to the relaxation parameter, the symbol $\mathbf{V}=\left(u\left(y,t\right),0\right)$ refers to convection velocity which is obtained by Poiseuille flow, $P\left(x,y,t\right)$ denotes a distribution function at the special location $\left(x,y\right)$ and time t, D_x and D_y are the diffusion coefficients along the x direction and y direction, respectively.

This model well overcomes the shortcomings of the Fick's law (contradicts the principle of causality [11, 12]) and Cattaneo model (violates material invariant formulation [13]). Recently, it attracts a large number of scholars' attention to study and analyze. Ciarletta and Straughan [14] analyzed the uniqueness and structural stability for the Cattaneo-Christov equations, results showed that the solution to the backward in time problem depended continuously on relaxation time. Based on the Cattaneo-Christov constitutive equation, Straughan [15] studied the thermal convection in a horizontal layer of incompressible Newtonian fluid with gravity acting downward, the paper showed that the thermal relaxation effect was significant and the