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DELAY-DEPENDENT STABILITY OF LINEAR MULTISTEP METHODS FOR NEUTRAL SYSTEMS WITH DISTRIBUTED DELAYS*

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Abstract

This paper considers the asymptotic stability of linear multistep (LM) methods for neutral systems with distributed delays. In particular, several sufficient conditions for delay-dependent stability of numerical solutions are obtained based on the argument principle. Compound quadrature formulae are used to compute the integrals. An algorithm is proposed to examine the delay-dependent stability of numerical solutions. Several numerical examples are performed to verify the theoretical results.

Mathematics subject classification: 65L05, 65L07, 65L20.
Key words: Neutral systems with distributed delays, Linear multistep methods, Delay-dependent stability, Argument principle.

1. Introduction

In this paper, we investigate the asymptotic stability of numerical methods for neutral systems with distributed delays which belongs to neutral delay integro-differential equations (NDIDEs)

$$\begin{cases} \dot{x}(t) = Lx(t) + Mx(t-\tau) + N\dot{x}(t-\tau) \\ \qquad\qquad + \displaystyle\int_{-\tau}^{0} K(s)x(t+s)ds + \int_{-\tau}^{0} R(s)\dot{x}(t+s)ds, \quad t > 0, \\ x(s) = \varphi(s), \qquad\qquad\qquad\qquad\qquad\qquad -\tau \le s \le 0 \end{cases} \tag{1.1}$$

with the condition

$$\|N\| + \int_{-\tau}^{0} \|R(s)\|ds \le \alpha < 1, \tag{1.2}$$

where $x(t) \in R^d$ is an unknown vector, parameter matrices $L, M, N, K(s), R(s) \in R^{d\times d}$ and delay τ is a positive number. Here $\varphi(t) \in C^1(-\tau, 0]$, the entries $k_{ij}(s)$ of the matrix $K(s)$ and the entries $r_{ij}(s)$ of the matrix $R(s)$ are continuous on $[-\tau, 0]$. NDIDEs plays an central role in a wide variety of scientific and technological fields, such as economics, population dynamics,

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control theory and so on (see, e.g., [3, 11, 17–19]), and hence has come to intrigue researchers in numerical computation and analysis (see, e.g., [5, 6, 9, 12, 13]).

In recent years, there is a growing interest in the stability analysis of numerical methods for NDIDEs. Zhao et al. [31] studied the stability of linear θ-method and BDF method for linear NDIDEs. Yu et al. [26], Zhang and He [27] investigated the stability of the numerical solution derived from Runge-Kutta methods and one-leg methods for nonlinear NDIDEs of the "Hale's form", respectively. Wang et al. [22] obtained nonlinear stability conditions for the neutral multidelay-integro-differential equations (NMIDEs). However, the stability region in the above research is independent of the delay term and we call it delay-independent stability. On the contrary, the stability which has relationship with delays is referred as delay-dependent stability and the stability analysis is much more difficult [2,15,16,24,25,30]. Wu and Gan [24] discussed the delay-dependent stability for the real coefficient linear test equations for NDIDEs. Zhang and Vandewalle [28] constructed the stability criteria for the asymptotic stability of Runge-Kutta and linear multistep methods for NMIDEs. Zhao et al. [29] analyzed the delay-dependent stability region of symmetric boundary value methods for the linear NDIDEs with four parameters. Then, Zhao et al. [30] derived the delay-dependent stability region of symmetric Runge-Kutta methods for NDIDEs. Up to now, few results on the delay-dependent stability for vector NDIDEs (1.1) have been presented in the literature.

It is well-known that the definition of D-stability, which is a kind of delay-dependent stability of numerical solutions for the delay differential equations given in literature [2, 10, 21], is too restrictive. For example, no A-stable natural Runge-Kutta methods for delay differential equations is D-stable. As a result, Hu and Mitsui [14] recently gave a novel definition for delay-dependent stability of numerical solutions referred as weak delay-dependent stability, which only requires that the difference scheme generated by a numerical method with a certain integer m arising in the stepsize $h = \tau/m$ is asymptotically stable. That is, the numerical solution is asymptotically stable, so long as there exists a natural number m for the stepsize $h = \tau/m$ which generates an asymptotically stable numerical solution. Furthermore, Hu and Mitsui obtained some sufficient conditions of delay-dependent stability of Runge-Kutta and linear multistep methods for delay differential equations of neutral type. In this paper, we focus our attention on the weak delay-dependent stability of linear multistep methods for the system (1.1) with condition (1.2).

Remark 1.1. When $N = 0$ and $R(s) = 0$, the system (1.1) degenerates into delay integro-differential equations (DIDEs). For the delay-dependent stability of numerical methods for DIDEs, one can refer to [8, 23].

The outline of the rest of the paper is as follows. First, several definitions and lemmas are reviewed in Section 2. Then, some new sufficient criteria of weak delay-dependent stability for linear multistep methods are suggested in Section 3. Numerical examples in Section 4 are presented to demonstrate the effectiveness of the theoretical results.

2. Preliminaries

In this section, we recall several definitions and lemmas which play a central role in the succeeding sections.

Now we introduce some notations. Throughout this paper, for a complex z, $\operatorname{Re} z$ and $\operatorname{Im} z$ denotes the real and imaginary parts of z, respectively. I stands for identity matrix, $\|Q\|$ means