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SIMULTANEOUSLY IMAGING AN INHOMOGENEOUS CONDUCTIVE MEDIUM AND VARIOUS IMPENETRABLE OBSTACLES*

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Abstract

Consider the inverse scattering of time-harmonic acoustic waves by a mixed-type scatterer consisting of an inhomogeneous penetrable medium with a conductive transmission condition and various impenetrable obstacles with different kinds of boundary conditions. Based on the establishment of the well-posedness result of the direct problem, we intend to develop a modified factorization method to simultaneously reconstruct both the support of the inhomogeneous conductive medium and the shape and location of various impenetrable obstacles by means of the far-field data for all incident plane waves at a fixed wave number. Numerical examples are carried out to illustrate the feasibility and effectiveness of the proposed inversion algorithms.

Mathematics subject classification: 35R30, 35Q60, 35P25, 78A46.
Key words: Inverse acoustic scattering, Modified factorization method, Numerical reconstruction, Inhomogeneous medium.

1. Introduction

In this paper, we study the inverse problem of reconstructing a mixed-type scatterer from the far-field measurements produced by all the incident plane waves at a fixed wave number. The scatterer is supposed to be the union of an inhomogeneous medium with the conductive transmission condition and different kinds of impenetrable obstacles. This problem occurs in lots of application areas such as radar and sonar, medical imaging and non-destructing testing, etc. Precisely, let an open bounded obstacle D_1 denote the inhomogeneous penetrable medium with a C^2-smooth boundary ∂D_1 and an open bounded obstacle D_2 denote the impenetrable obstacle with a C^2-smooth boundary ∂D_2. Denote by $D_0 := \mathbb{R}^n\backslash(\overline{D}_1 \cup \overline{D}_2)$ (where $n = 2, 3$, for convenience, we will consider the case when $n = 3$) which is connected. We further assume that $D_1 \cap D_2 = \emptyset$ (see Fig. 1.1 for the geometric configuration of the mixed scattering problem).

Suppose that D_1 is filled with an inhomogeneous material characterized by $n(x)$, which is known as the refractive index satisfying that $n(x) \in L^\infty(\mathbb{R}^3)$ with $\mathrm{Re}[n(x)] < 1$ and $\mathrm{Im}[n(x)] \geq c_0 > 0$ with a positive constant c_0, whereas the exterior part D_0 is filled with a homogeneous material with the refractive index $n(x) = 1$. For simplicity, we only consider the case when an impedance boundary condition is imposed on ∂D_2. The same results can be similarly extended to the other cases, e.g. the Dirichlet or the Neumann boundary condition on ∂D_2.

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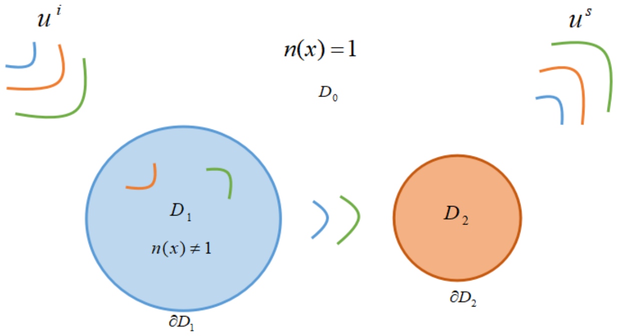

Fig. 1.1. Graphical representation of the mixed scattering problem.

Consider the incident wave field $u^i = e^{ikx\cdot d}$ with the wave number $k > 0$ and the incident direction $d \in \mathbb{S}^2$. Then scattering of time-harmonic acoustic waves by the mixed-type scatterer can be modeled by the following Helmholtz equation with a conductive transmission boundary condition on ∂D_1 and an impedance boundary condition on ∂D_2:

$$\begin{cases} \Delta u + k^2 u = 0 & \text{in } D_0, \\ \Delta v + k^2 n(x) v = 0 & \text{in } D_1, \\ u - v = 0 & \text{on } \partial D_1, \\ \dfrac{\partial u}{\partial \nu} - \dfrac{\partial v}{\partial \nu} + \mu u = 0 & \text{on } \partial D_1, \\ \dfrac{\partial u}{\partial \nu} + i\lambda u = 0 & \text{on } \partial D_2. \end{cases} \tag{1.1}$$

Here ν is the unit normal on ∂D_1 directed into $\mathbb{R}^3\backslash\overline{D}_1$, and on ∂D_2 directed into $\mathbb{R}^3\backslash\overline{D}_2$, respectively, and μ is the constant conductivity parameter satisfying that $\mathrm{Re}(\mu) < 0, \mathrm{Im}(\mu) \geq \mu_0 > 0, \lambda > 0$ is a positive constant, and $u = u^i + u^s$ denotes the total field in D_0 and $v = u^i + v^s$ denotes the total field in D_1 with the incident wave $u^i = e^{ikx\cdot d}$ and the scattered fields u^s and v^s, respectively. Moreover, the scattered field u^s satisfies the Sommerfeld radiation condition

$$\frac{\partial u^s}{\partial |x|} - iku^s = \mathcal{O}\left(\frac{1}{|x|^2}\right) \quad \text{as} \quad |x| \to \infty. \tag{1.2}$$

It is well-known that the scattered field u^s has the asymptotic behavior [6]

$$u^s(x) = \frac{e^{ik|x|}}{4\pi|x|} u_\infty(\widehat{x}) + \mathcal{O}\left(\frac{1}{|x|^2}\right) \quad \text{as} \quad |x| \to \infty, \tag{1.3}$$

uniformly for all $\widehat{x} = x/|x|$, where u_∞ is known as the far-field pattern of u^s, which is an analytic function defined on $\mathbb{S}^2$.

The well-posedness of the scattering problem (1.1)-(1.2) can be established by applying the variational method (see also [15, 23]). In the current paper, we are interested in the inverse problem of simultaneously reconstructing the shape and location of the inhomogeneous penetrable medium D_1 and the impenetrable obstacle D_2 from a knowledge of the far-field pattern u_∞ for all incident plane waves at a fixed frequency by using a modified factorization method. The factorization method was first introduced by Kirsch [9] for the Dirichlet scattering problem. We also refer the readers to the monographs [3, 12] for a comprehensive account on the