Journal of Computational Mathematics
Vol.43, No.4, 2025, 840–865.
http://www.global-sci.org/jcm
doi:10.4208/jcm.2402-m2023-0194

TAMED STOCHASTIC RUNGE-KUTTA-CHEBYSHEV METHODS FOR STOCHASTIC DIFFERENTIAL EQUATIONS WITH NON-GLOBALLY LIPSCHITZ COEFFICIENTS*

Yanyan Yu, Aiguo Xiao and Xiao Tang[1)]
School of Mathematics and Computational Science, Xiangtan University, Xiangtan 411105, China
Emails: 202131510104@smail.xtu.edu.cn, xag@xtu.edu.cn, tangx@xtu.edu.cn

Abstract

In this paper, we introduce a new class of explicit numerical methods called the tamed stochastic Runge-Kutta-Chebyshev (t-SRKC) methods, which apply the idea of taming to the stochastic Runge-Kutta-Chebyshev (SRKC) methods. The key advantage of our explicit methods is that they can be suitable for stochastic differential equations with non-globally Lipschitz coefficients and stiffness. Under certain non-globally Lipschitz conditions, we study the strong convergence of our methods and prove that the order of strong convergence is 1/2. To show the advantages of our methods, we compare them with some existing explicit methods (including the Euler-Maruyama method, balanced Euler-Maruyama method and two types of SRKC methods) through several numerical examples. The numerical results show that our t-SRKC methods are efficient, especially for stiff stochastic differential equations.

Mathematics subject classification: 65C30, 60H10, 60H35.
Key words: Stochastic differential equation, Non-globally Lipschitz coefficient, Stiffness, Explicit tamed stochastic Runge-Kutta-Chebyshev method, Strong convergence.

1. Introduction

Stochastic differential equations (SDEs) have a wide range of applications in simulating problems across various fields such as biology, chemistry, physics, and economics [6, 16, 21, 22, 28, 29]. Since obtaining analytical solutions of most SDEs directly is often difficult, the development of effective numerical methods for approximating stochastic equations has become the focus of research. Under the classical assumptions that drift and diffusion coefficients meet the global Lipschitz condition and linear growth condition, there have been many significant research results about the numerical solutions of SDEs. The most commonly used methods include Euler-Maruyama (EM) method [18], Milstein method [19], etc.

However, the coefficients of many important SDEs do not satisfy the global Lipschitz condition. This leads to the divergence of traditional explicit numerical methods, see e.g. [7, 20]. To overcome this difficulty, many modified explicit methods for the SDEs with non-globally Lipschitz continuous coefficients have been proposed, see e.g. [11–13, 17]. In particular, the balanced and tamed explicit methods have received much attention from scholars in recent years, for example, the balanced EM method [24], the balanced Milstein method [31], the split-step balanced θ-method [14], the semi-tamed Milstein method [15], the tamed Milstein method [27], and the tamed Runge-Kutta methods [4].

* Received September 8, 2023 / Revised version received December 18, 2023 / Accepted February 29, 2024 / Published online May 24, 2024 /
1) Corresponding author

In fact, the SDEs from application problems may not only have coefficients which do not meet global Lipschitz condition, but may also be stiff, for example, see [9]. Although the modified explicit numerical methods proposed in [13–15, 17, 27, 31] can handle some situations where the coefficients do not satisfy the global Lipschitz condition, they are still not suitable for stiff problems. To handle stiff terms, implicit numerical methods are often used. However, when the dimensions of the equations are high and the problems are complex, it is difficult to implement the implicit methods. This limits the practicality of the implicit methods.

For ordinary differential equations with mild stiffness, the explicit Runge-Kutta-Chebyshev (RKC) methods are relatively suitable for solving such equations because of their long stability domain along the negative real axis, see e.g. [25,26]. Some scholars have extended these methods to stiff SDEs and constructed the corresponding stochastic Runge-Kutta-Chebyshev (SRKC) methods, as shown in [1,3,10]. These SRKC methods are able to inherit the advantage of RKC methods in stability. However, it should be noted that these methods can only ensure strong convergence under the global Lipschitz condition. In fact, when the equation coefficients fail to satisfy the global Lipschitz condition, these methods do not converge in some cases (see the numerical experiments section).

In the present paper, we apply the taming technique to the SRKC methods and propose the tamed stochastic Runge-Kutta-Chebyshev (t-SRKC) methods. Under certain non-globally Lipschitz conditions, the strong convergence of our t-SRKC methods is proved, and the strong convergence order is 1/2. On the one hand, our t-SRKC methods have a wider range of applications than the traditional SRKC methods proposed in [1, 3, 10] because our t-SRKC methods are applicable to some problems with non-globally Lipschitz coefficients. On the other hand, for the stiff SDEs, our t-SRKC methods can avoid the step size restriction problem which the existing balanced or tamed explicit methods (such as the balanced EM method in [24]) suffer from. Finally, the numerical results well verify the advantages of our t-SRKC methods in the above two aspects.

The remaining sections of this paper are structured as follows. In Section 2, we introduce some useful assumptions and conclusions. In Section 3, we propose the t-SRKC methods (3.10). We discuss the boundedness of moments for the t-SRKC methods (3.10) in Section 4. In Section 5, we study the strong convergence of the t-SRKC methods (3.10). Finally, we present our numerical results in Section 6.

2. Preliminary

Let $(\Omega, \mathcal{F}, P)$ be a complete probability space and $\mathcal{F}_t^W$ be an increasing family of σ-subalgebras of $\mathcal{F}$ induced by $W(t)$ with $0 \le t \le T$, where $W(t) = (W_1(t), \ldots, W_m(t))^\top$ is an m-dimensional standard Wiener process. Let $|\cdot|$ denote the Euclidean norm on $\mathbb{R}^d$. In this paper, we study the following system of Itô SDEs:

$$dX(t) = f\big(t, X(t)\big)dt + \sum_{r=1}^{m} g_r\big(t, X(t)\big)dW_r(t), \quad t \in (t_0, T], \quad X(t_0) = X_0, \tag{2.1}$$

where X, f, g_r are d-dimensional column-vectors. We suppose that the solution $X_{t_0,X_0}(t)$ of (2.1) is well-defined on $[t_0, T]$.

We consider the equidistant discretization $\mathcal{T}_h : t_0 < t_1 < t_2 < \cdots < t_N = T$ of the time interval $[t_0, T]$ with $t_j = t_0 + jh, h = (T - t_0)/N$ for $j = 0, 1, \ldots, N$. For convenience, we use C to represent a generic constant, which may represent different values in different places.