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DIFFUSE OPTICAL TOMOGRAPHY IN TIME DOMAIN WITH THE INVERSE RYTOV SERIES*

Chi Zhang
Department of Cellular and Molecular Anatomy, Hamamatsu University School of Medicine, Hamamatsu 431-3192, Japan
Manabu Machida[1)]
Department of Informatics, Faculty of Engineering, Kindai University, Higashi-Hiroshima 739-2116, Japan
Email: machida@hiro.kindai.ac.jp

Abstract

The Rytov approximation has been commonly used to obtain reconstructed images for optical tomography. However, the method requires linearization of the nonlinear inverse problem. Here, we demonstrate nonlinear Rytov approximations by developing the inverse Rytov series for the time-dependent diffusion equation. The method is verified by a solid-phantom experiment.

Mathematics subject classification: 65M32, 78A46.
Key words: Inverse problems, Diffuse optical tomography.

1. Introduction

Optical tomography obtains reconstructed images similar to the X-ray computed tomography [7]. However, the inverse problem for optical tomography becomes nonlinear and severely ill-posed because near-infrared light, which is used for optical tomography, is multiply scattered in biological tissue [2]. One way to obtain reconstructed images of optical tomography is to solve the minimization problem for a cost function by an iterative scheme. Such iterative methods do not work especially for clinical research, in which less a priori knowledge is available compared with phantom experiments; choosing a good initial guess is difficult and the calculation is trapped by a local minimum since the cost function of optical tomography has a complicated landscape with local minima. The other way is to directly reconstruct perturbation of a coefficient. The Born and Rytov approximations are known in the direct approach. The Rytov approximation has been used in practical situations including optical tomography for the breast cancer [8] and brain function [11]. It was numerically demonstrated that the Rytov approximation appeared superior [3]. The drawback of the (first) Born and Rytov approximations is that these methods require linearization of nonlinear inverse problems. That is, nonlinear terms in the Born and Rytov series are ignored.

In this paper, we will develop the latter approach of perturbation and consider nonlinear Rytov approximations. Although the Rytov approximation has been commonly used in optical tomography, it was only recently devised how to solve inverse problems by taking nonlinear terms in the Rytov series into account [18].

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1) Corresponding author

The inverse Born series has been developed to invert the Born series [27]. The inverse Born series was considered for the Helmholtz equation [32], the diffusion equation [22, 23], and the inverse scattering problem [28]. Its mathematical properties and recursive algorithm were developed [25, 26]. Furthermore, the inverse Born series was studied for the Calderón problem [4], scalar waves [14], the inverse transport problem [20], electromagnetic scattering [15], discrete inverse problems [9], and the Bremmer series [31]. In [6,17], the inverse Born series was extended to Banach spaces. In [1], a modified Born series with unconditional convergence was proposed and its inverse series was studied. In [12], the convergence theorem for the inverse Born series has recently been improved. A reduced inverse Born series was proposed [24]. The inverse Born series was extended to a nonlinear equation [10]. Its convergence, stability, and approximation error were proved under H^s norm [21].

The comparison of the Born and Rytov approximations has been discussed [13, 16]. It is known that better reconstructed images can be obtained by the Rytov approximation. To extend the Rytov approximation, the inversion of the Rytov series has been studied. In [33], the inversion for the Helmholtz equation was performed but no general way of considering nonlinear terms was obtained. In [29], the inversion of the Rytov series was studied but each term in the obtained series contains infinitely many higher-order terms and numerical reconstruction based on the obtained series was not feasible. In [18], the inverse Rytov series was constructed to invert the Rytov series. Each term in the inverse Rytov series can be recursively computed. In this paper, by developing [18], we will consider the inverse Rytov series for diffuse optical tomography in time domain and furthermore verify the inverse series experimentally.

The rest of the paper is organized as follows. In Section 2, the diffuse light is expressed in the form of a series. In particular, the numerical algorithm for nonlinear Rytov approximations is explained in Section 2.2. In Section 3, the experimental setup is described. To handle experimental data, we consider another series by taking difference of the inverse series. Results are shown in Section 4. Section 5 is devoted to the concluding remarks.

2. Forward and Inverse Series for Diffuse Light

2.1. Time-dependent diffusion equation

Let Ω be the half-space in $\mathbb{R}^3$. The boundary of Ω is denoted by $\partial\Omega$. Let c be the speed of light in Ω. Let $x_s \in \Omega$ be the position of the source. The energy density $u(x, t; x_s)$ ($x \in \Omega$, $t \in (0, T)$) of near-infrared light in biological tissue is governed by the following diffusion equation [2, 5]:

$$\begin{cases} \partial_t u - D_0 \Delta u + \alpha u = S, & (x,t) \in \Omega \times (0,T), \\ u = 0, & (x,t) \in \partial\Omega \times (0,T), \\ u = 0, & x \in \Omega, \quad t = 0, \end{cases}$$

where $S(x, t)$ is the source term and $D_0 = c/(3\mu_s') > 0$ is the diffusion coefficient with the reduced scattering coefficient μ_s'. The positive constant $\alpha = c\mu_a$ is given by c and the absorption coefficient μ_a. Let $\delta(\cdot)$ be Dirac's delta function. The source term is given by

$$S(x, t) = g\delta(x - x_s)h(t),$$

where $h(t)$ is the temporal profile of the light source, $g > 0$ is a constant. We suppose that measurements are performed at M_{SD} places $x_s = x_s^{(i)} = (x_{s1}^{(i)}, x_{s2}^{(i)}, \ell), i = 1, \ldots, M_{\mathrm{SD}}$ on $\partial\Omega$.