Journal of Computational Mathematics
Vol.44, No.2, 2026, 521–538.
http://www.global-sci.org/jcm
doi:10.4208/jcm.2502-m2024-0208

GENERALIZED JACOBI SPECTRAL GALERKIN METHOD FOR FRACTIONAL-ORDER VOLTERRA INTEGRO-DIFFERENTIAL EQUATIONS*

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Abstract

In this paper, we present a generalized Jacobi spectral Galerkin method for fractional Volterra integro-differential equations (FVIDEs). The basis functions of the proposed method are generalized Jacobi functions, which serve as natural basis functions for appropriately designed spectral methods for FVIDEs. We establish a convergence analysis of the generalized Jacobi spectral Galerkin method under reasonable assumptions. Numerical experiments are provided to demonstrate the effectiveness of the proposed method.

Mathematics subject classification: 65L05, 65L20, 65L50.
Key words: Fractional-order Volterra integro-differential equations, Spectral Galerkin method, Generalized Jacobi function.

1. Introduction

The fractional calculus has gained a tremendous development in both theory and application, and its appearance and development have made up for defects of the classical calculus of integer-order to a certain extent. Progress in the last two decades has demonstrated that fluid flow in porous materials, anomalous diffusion transport, acoustic wave propagation in viscoelastic materials, dynamics in self-similar structures, signal processing, financial theory, electric conductance of biological systems are more accurately described by involving fractional calculus (cf. [11, 17, 20, 25, 28] and the references therein).

Fractional integral equations and integro-differential equations are usually very difficult to solve analytically and the exact solutions to this type of problem are very scarce. To solve these problems numerically, a great deal of interest has been emphasized by several researchers. In the last thirty years, there have been a great deal of methods to solve fractional-order integro-differential equations numerically, for instance Adomian decomposition method (ADM) [22],

* Received September 20, 2024 / Revised version received December 19, 2024 / Accepted February 17, 2025 / Published online March 24, 2025 /
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fractional differential transform method (FDTM) [23], the collocation method [30], variational iteration method (VIM) [3], Laplace transform method [27] and homotopy analysis method [15].

This paper is concerned with the numerical solutions of the fractional-order Volterra integro-differential equations

$$\begin{cases} {}_0D_t^{\mu} y(t) = y(t) + \int_0^t K(t,\tau)y(\tau)d\tau + g(t), \quad t \in I := [0,T], \\ y(0) = y_0, \end{cases} \tag{1.1}$$

where $0 < \mu < 1$, $K \in C(D)$ with

$$D := \{(t,\tau) : 0 \leqslant \tau \leqslant t \leqslant T\}, \quad g(t) \in C(I).$$

${}_0D_t^{\mu}$ denotes the left-sided Reimann-Liouville fractional derivative of order μ.

Many mathematical modeling of various physical phenomena contains fractional-order Volterra integro-differential equations, for instance, heat conduction in materials with memory in [24]. In addition, such kinds of equations are also encountered in conduction, convection and radiation problems (see, e.g. [1,5,26] and the references therein).

In recent years, more and more researchers have made contributions to the fractional-order Volterra integro-differential equations. Yang [39] proposed general spectral and pseudo-spectral Jacobi-Galerkin methods for fractional-order integro-differential equations of Volterra type. Zhu and Fan [42] proposed a second Chebyshev wavelet method (SCWM) to solve nonlinear fractional-order Volterra integro-differential equations. In [13], the Legendre wavelet method (LWM) has been applied to solve the fractional population growth model which is the form of the fractional Volterra-Fredholm integro-differential equation (FVFIDE). The nonlinear FVFIDEs with mixed conditions have been solved by Nystrom and Newton-Kantorovitch method. In addition, orthogonal polynomial approximations have been applied to fractional integro-differential equations, viz., Bernoulli polynomials [35], least squares method and shifted Chebyshev Polynomial [21], Bernstein operational matrix [2], etc.

The spectral Galerkin method is an efficient method that has been applied by many researchers to solve different types of partial integro-differential equations. In [32], a generalized Jacobi-Galerkin method is applied to nonlinear Volterra integral equations with weakly singular kernels. Xie *et al.* [38] proposed Jacobi spectral Galerkin approaches for the second kind Volterra integral equation. Wan *et al.* [36] also proposed Legendre spectral Galerkin method for second kind VIEs. Yang [40] employed a Jacobi spectral Galerkin method for VIEs with a weakly singular kernel. In addition to this, Petrov-Galerkin method has been successively applied to solve many partial differential equations (see, e.g. [16,31]).

When $\mu = 1$, the Eq. (1.1) is the classical Volterra integro-differential equations (VIDEs)

$$\begin{cases} y'(t) = y(t) + \int_0^t K(t,\tau)y(\tau)d\tau + g(t), \quad t \in [0,T], \\ y(0) = y_0. \end{cases} \tag{1.2}$$

Recently, many kinds of spectral collocation methods are proposed for solving VIEs with smooth kernels (cf. [9,18,34,37] and the references therein). To solve VIEs with weakly singular kernels, many attempts have been made to overcome the difficulties caused by the singularities of the solutions. Chen and Tang [7,8] proposed spectral collocation methods for weakly singular VIEs. In [19], linear Volterra integro-differential equations have been solved by Petrov-Galerkin