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GREEDY LOCAL REFINEMENT FOR ANALYSIS-SUITABLE T-SPLINES WITH LINEAR COMPLEXITY*

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Abstract

Achieving linear complexity is crucial for demonstrating optimal convergence rates in adaptive refinement. It has been shown that the existing linear complexity local refinement algorithm for T-splines generally produces more degrees of freedom than the existing greedy refinement, which lacks linear complexity. This paper introduces a novel greedy local refinement algorithm for analysis-suitable T-splines, which achieves linear complexity and requires fewer control points than existing algorithms with linear complexity. Our approach is based on the observation that confining refinements around each T-junction to a pre-established feasible region ensures the algorithm's linear complexity. Building on this constraint, we propose a greedy optimization local refinement algorithm that upholds linear complexity while significantly reducing the degrees of freedom relative to previous linear complexity local refinement methods.

Mathematics subject classification: 65D07.
Key words: T-splines, Analysis-suitable T-splines, Linear complexity, Isogeometric analysis.

1. Introduction

Isogeometric analysis (IGA) is a potent numerical method for solving partial differential equations (PDEs) [6, 16, 26, 27, 29]. The basic idea of IGA uses the spline functions from computer aided design (CAD) for numerical simulation, eliminating the necessity for model conversion. In the early stages of IGA research, the most commonly used representation is non-uniform rational B-splines (NURBS). However, the tensor-product structure makes knot insertion having global impacts. In order to introduce adaptive refinement into NURBS representation, several relevant constructions are receiving particular attention, such as hierarchical B-splines [10, 14], T-splines [30, 32], PHT-splines [7, 19] and LR splines [8].

Among these technologies, hierarchical B-splines and T-splines are the two most used ones. Hierarchical B-splines [10] represent a classic local refinement method, ensuring the nested spaces and the linear independence of the basis functions [17]. Truncated basis for hierarchical splines (THB-splines) has been introduced [12, 13], which leads to more localized and unstructured meshes. Adaptive isogeometric methods with hierarchical splines [3–5] have linear complexity and optimal convergence rates.

T-splines [30, 32] are an important technology in industrial design [31] and isogeometric analysis [15, 23, 28, 29, 34]. Although the whole class of T-splines is not suitable as a basis for

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IGA because of possible linear dependence [2], analysis-suitable T-splines (AST-splines), are defined [20–22, 28, 33] to satisfy all the desirable properties for IGA.

The achievement of linear complexity plays a pivotal role in establishing the optimal convergence rates of adaptive refinement algorithms [9]. Firstly, Scott *et al.* [28] introduced a local refinement algorithm for AST-splines based on a greedy strategy. However, its theoretical proof of possessing linear complexity remains elusive. In contrast, the refinement algorithms developed by [11, 25] guarantee linear complexity and optimal convergence rates. Nevertheless, this approach leads to a substantial increase in the degrees of freedom associated with AST-splines.

Our objective is to synergize the advantages of two distinct algorithms, thereby reducing the degrees of freedom while maintaining linear complexity. The main idea is to select only some of the additional edges of the algorithm from [25] using an iterative greedy-algorithm based on the assignment of a weight to each feasible edge insertion. Notably, we transform the nested neighborhoods, originally discussed by [25], into distance constraints between edges. This modification facilitates direct input of complex initial meshes without the need for repeated iterative computations. By integrating these new distance constraints with the greedy strategy, we achieve a significant reduction in the degrees of freedom within the resulting AST-mesh. This enhancement enhances the practical applicability of our local refinement algorithm.

This paper is organized as follows. T-splines and AST-splines are reviewed in Section 2. Section 3 details the greedy local refinement algorithm. Section 4 demonstrates the superiority of the algorithm through numerical experiments. The last section presents the conclusion and future work.

2. T-splines

In this section, we review the basic concepts of T-meshes and T-splines [1].

2.1. Index T-meshes

Following the approach proposed by [1], we define T-splines based on T-meshes within the index domain, referred to as index T-meshes in this paper. A T-mesh $\mathcal{T}$ for a bi-degree (d_1, d_2) T-spline consists of faces, edges, and vertices that form a rectangular partition of the index domain $[0, c + d_1] \times [0, r + d_2]$. Each corner (or vertex) of the rectangles is located at integer coordinates. Let $p = \lfloor (d_1 + 1)/2 \rfloor$ and $q = \lfloor (d_2 + 1)/2 \rfloor$, which represent the maximal integers less than or equal to $(d_1 + 1)/2$ and $(d_2 + 1)/2$, respectively. Then the active region is denoted as the rectangular region $[p, c + d_1 - p] \times [q, r + d_2 - q]$. As elucidated below, the active region encompasses the anchors pertinent to the blending functions, while other indices are required for defining the blending function when the anchor is close to the boundary.

A T-mesh consists of three fundamental elements: vertices, edges, and faces. A vertex, representing the corner of a rectangle within the T-mesh, can be denoted as (σ_i, τ_i) or $\{\sigma_i\} \times \{\tau_i\}$. An edge, characterized as a line segment connecting two vertices without any intermediate vertices, is represented by $[\sigma_j, \sigma_k] \times \{\tau_i\}$ or $\{\sigma_i\} \times [\tau_j, \tau_k]$ for horizontal or vertical edges, respectively. A face, defined as a rectangle devoid of any internal edges or vertices, is represented by $[\sigma_i, \sigma_j] \times [\tau_i, \tau_j]$ or $(\sigma_i, \sigma_j) \times (\tau_i, \tau_j)$. The valence of a vertex is the number of edges that have the vertex as an endpoint. For interior vertices, only I-junctions, valence three (called T-junctions), or valence four vertices are allowed. The symbols $\vdash$, $\dashv$, $\perp$, and $\top$ denote the four potential orientations of T-junctions. A T-mesh $\mathcal{T}$ for a bi-degree (d_1, d_2) T-spline is admissible if for any vertex (i, j), if $0 \leq i \leq p$ or $c + d_1 - p \leq i \leq c + d_1$, the vertex cannot be $\perp, \top$ and if