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A CDM-Based Tensile Instability Analysis in DP1180 Steel Sheet

Bingying Zhou[1], Xuexi Cui[2], Bobin Guan[3], Zhigang Liu[4], Fei Han[5], Min Wan[1] and Xiangdong Wu[1,*]

[1] *School of Mechanical Engineering and Automation, Beihang University, Beijing 100191, China*
[2] *Beijing Institute of Aerospace Testing Technology, Beijing 100074, China*
[3] *China Academy of Launch Vehicle Technology, Beijing 100076, China*
[4] *Institute of High Performance Computing, A*STAR, 1 Fusionopolis Way, 138632 Singapore*
[5] *R & D Center, Baoshan Iron & Steel Co., Ltd., Shanghai 201900, China*

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Abstract. Plastic instability is an inherent property of metal materials, and enhancing the formability of sheet metal can be attributed to the retardation of plastic instability. In this study, we revisited the tensile instability of dual-phase (DP) steel by introducing a novel damage evolution model. Leveraging the theory of plastic instability and continuum damage mechanics (CDM), the instabilities of DP1180 steel sheet were investigated by examining the rotation angle of the uniaxial tensile specimen, which serves as an indicator for the onset of localized necking instability. Based on the hypothesis of elastic modulus equivalence, the damage evolution equation in uniaxial tension was derived mathematically, and the implicit and explicit expressions of the damage evolution were proposed. Furthermore, the relationship between effective stress and equivalent strain in the process of uniaxial tensile test was obtained. It was found that the sharp increase of effective stress was the direct factor leading to the fracture of DP1180 steel sheet. Finally, the mechanism of damage evolution was studied by scanning electron microscopy (SEM), including the nucleation, growth, and coalescence of voids. This study elucidates the mechanisms of tensile instabilities in DP steel sheets and provides a potential damage criterion for localized instability.

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*Corresponding author.
Email: wuxiangdongbuaa@163.com (X. Wu)

1 Introduction

Sheet metal forming is an important manufacturing process that is widely used in aerospace, automotive, and other fields. However, the plastic instability, which is an inherent property of ductile metal materials, must occur in the later stage of sheet metal forming. To accurately quantify the formability, the plastic instability has been attracted extensive attention. Plastic instability can be categorized as diffuse instability and localized instability [1]. While the load reaches maximum, the sheet is in a state of diffuse instability, which was proposed by Swift [2]. Localized instability was proposed by Hill [3]. For the sheet usually has considerable deformability after diffuse necking, the localized instability has attracted more attention and been regarded as the forming limit criterion, which is vital to quantitatively evaluate the formability of sheet under complex loading conditions.

Chen and Hu [4,5] found that the strain state gradually changes to plane strain state whatever the initial strain state was, and proposed an instability model, which is called C-H model. Hora et al. [6,7] explained the change of strain state from the point of yield loci and modified the maximum force criterion, which is called MMFC model. It has been widely accepted to determine the localized instability as well as the forming limit curve (FLC) according to the transition to plane strain state [8–10]. The above instability theories assume that material has perfect homogeneity, which means that the effect of damage is not considered. However, the failure of ductile material is generally attributed to the accumulation of damage. Based on this view, Marciniak and Kuczyński [11] developed an instability theory to predict the FLC based on the assumption: the localized necking is caused by the thickness and/or microstructural non-homogeneity. The M-K model and its modifications have been used and studied extensively [12–14]. Basak et al. [15,16] analyzed the influence of anisotropy on the prediction accuracy of localized instability based on Yld2000-2d and M-K model. Although classical instability theory is well-established and can predict the instability behavior of sheet metal effectively, it requires highly accurate material property parameters such as the strain hardening index (n) and the anisotropy index (r). The accuracy of these parameters directly affects the prediction of localized instability. However, obtaining these accurate property parameters from experiments is challenging, especially when they evolve with plastic strain, which complicates the application of instability theory.

The development of digital image correlation (DIC) provides an effective way to get the evolution of full-field strain, based on which, three kinds of methods to obtain the point of localized instability, namely spatial method, temporal method, and spatiotemporal method, were proposed [17,18]. The spatial method is based on the strain distribution on the surface of specimen after local necking [19–21]. The temporal methods are by calculating the derivative of strain concerning time [22–25]. The spatiotemporal method mainly combines the spatial and temporal methods [17,26]. Recently, Fang et al. [27] proposed a new method for localized necking detection in uniaxial tensile testing based on a multi-camera DIC system which belongs to the spatiotemporal method. Apart from