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HAM-Schrödingerisation: A Generic Framework of Quantum Simulation for Any Nonlinear PDEs

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Abstract. Recently, Jin et al. proposed a quantum simulation technique for **any** *linear* partial differential equations (PDEs), called Schrödingerisation [1–3]. In this paper, the Schrödingerisation technique for quantum simulation is expanded to **any** *nonlinear* PDEs by combining it with the homotopy analysis method (HAM) [4–6]. The HAM can transfer a nonlinear PDE into a series of linear PDEs with guaranteeing convergence of the series. In this way, **any** nonlinear PDEs can be solved by quantum simulation using a quantum computer. For simplicity, we call the procedure "HAM-Schrödingerisation quantum algorithm". Quantum computing is a groundbreaking technique. Hopefully, the "HAM-Schrödingerisation quantum algorithm" can open a door to highly efficient simulation of complicated turbulent flows by means of quantum computing in future.

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Key words: Quantum computing, homotopy analysis method, nonlinearity.

1 Introduction

Today, quantum computing [7–15] offers a core opportunity for computational methods. Hamiltonian simulation is likely to be of particular importance in quantum computing, and is valid for the following time-dependent Schrödinger equation

$$i\,\partial_t \psi = H(t)\,\psi, \tag{1.1}$$

where $H(t)$ is a time-dependent Hamiltonian operator, ψ is a function, t denotes the time, and $i=\sqrt{-1}$, respectively.

Jin et al. [1–3] introduced a generic framework, called Schrödingerisation, which can map *any* linear PDEs into Schrödinger equations in real time. This is a milestone in

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quantum simulation. Based on a new approach called "warped phase transformation", Schrödingerisation can be used to solve *any* system of linear PDEs using quantum simulation, where the general form of the PDEs is given by

$$\frac{\partial \psi(\mathbf{r},t)}{\partial t} = \mathcal{L}[\psi(\mathbf{r},t)] + f(\mathbf{r},t), \quad \mathbf{r} \in \Omega, \quad t \geq 0, \tag{1.2}$$

subject to the initial condition

$$\psi(\mathbf{r},0) = \alpha(\mathbf{r}) \tag{1.3}$$

and the boundary condition

$$\psi(\mathbf{r},t) = \beta(t), \quad \text{when } \mathbf{r} \in \Gamma, \tag{1.4}$$

where $\mathbf{r}$ and t denote the spatial and temporal variable, $\psi(\mathbf{r},t)$ is a unknown function, $\mathcal{L}$ is a linear operator, $f(\mathbf{r},t)$, $\alpha(\mathbf{r})$, and $\beta(t)$ are known functions, Ω denotes the physical domain and Γ denotes its boundary, respectively. In practice, a linear PDE in the form (1.2)–(1.4) can be discretized in space to get a system of linear ordinary differential equations (ODEs) as follows:

$$\frac{d\mathbf{u}(t)}{dt} = A(t)\mathbf{u}(t) + \mathbf{b}(t), \quad \mathbf{u}(0) = \mathbf{a}, \tag{1.5}$$

where $\mathbf{b} \in \mathbb{C}^n$ and $\mathbf{u} \in \mathbb{C}^n$ are known functions, $\mathbf{a} \in \mathbb{C}^n$ is a known vector, $A(t) \in \mathbb{C}^{n \times n}$ can be a non-Hermitian matrix, i.e., $A(t)$ might not be equal to its conjugate transpose, respectively. The key point is that the linear equation (1.5) can be solved using the quantum simulation technique Schrödingerisation [1–3]. Several applications are described in the literature illustrating the validity of Schrödingerisation in solving many types of linear PDEs [16–25].

An important question follows. Can any *nonlinear* PDEs be solved by means of quantum computing? The answer is yes, as shown in Section 2.

2 HAM-based quantum simulation for nonlinear PDEs

More specifically, can any nonlinear PDE be transferred into a series of linear PDEs with *convergence guarantee* of the solution series? The answer is yes, as described below.

Mechanics, as a significant branch of natural sciences, often deals with the core problem of solving nonlinear equations. To do a good job, one must first sharpen one's tools. Continuously breaking through the limitations of traditional methods and proposing more effective new approaches is one of the essential tasks in modern mechanics. Analytical solutions possess unique advantages, as they can uncover the universal laws and essential characteristics of a problem. Traditional analytical approximation methods, represented by perturbation methods, typically rely on small physical parameters and often encounter issues such as solution divergence or slow convergence, thus are generally only applicable to weakly nonlinear problems.