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PINN-Based Partial Learning for Multicontinuum Problems: Dual Continuum Fluid Filtration Simulation

Wanbao Huang[1], Yin Yang[1,*], Dmitry Ammosov[2], Wei Xie[1] and Sergei Stepanov[2]

[1] *School of Mathematics and Computational Science, Xiangtan University, National Center for Applied Mathematics in Hunan, Xiangtan, Hunan 411105, China*
[2] *Computational Technologies & Artificial Intelligence, North-Eastern Federal University, Yakutsk 677980, Russia*

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Abstract. In this paper, we consider the dual continuum fluid filtration problem. The mathematical model is described by a coupled system of partial differential equations for two pressure fields. We propose a novel PINN-based partial learning approach to solve this problem. We employ a partially explicit time scheme, where the implicit scheme is applied to a highly permeable continuum, and the explicit scheme is used for the low permeable continuum. We utilize discrete-time physics-informed neural networks (PINNs) to compute the implicit part of the problem that is challenging to solve directly. By leveraging the slow variations in the low permeable continuum and the robustness of PINNs to data noise, we decouple the equations and solve them sequentially. At each time layer, we first solve the implicit part and then treat it as a known function to solve the explicit part. The explicit part is spatially discretized using the finite element method with standard linear basis functions. We utilize transfer learning to accelerate the computation of the implicit part using the parameters from the previous time layer as the initial parameters for training the current time layer. To test the proposed method, we consider two two-dimensional model problems. For each problem, the method achieved good accuracy, with transfer learning significantly speeding up the computation.

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*Corresponding author.
Emails: huangwanbao@smail.xtu.edu.cn (W. Huang), yangyinxtu@xtu.edu.cn (Y. Yang), dmitryammosov@gmail.com (D. Ammosov), xiew@smail.xtu.edu.cn (W. Xie), cepe2a@inbox.ru (S. Stepanov)

1 Introduction

Fluid filtration, crucial for reservoir simulation [1–3], presents significant challenges in numerical simulation. One of the main challenges is the high heterogeneity of the medium properties in practical applications. Accurate modeling of such media requires using very fine computational grids, which increases computational cost. Various multiscale methods are widely used to reduce the computational resource requirements [4–15]. For problems requiring high-resolution approximations, spectral methods combined with nonlocal operator extensions offer an alternative pathway to balance accuracy and efficiency [16]. One of the most popular upscaling approaches in filtration is multicontinuum modeling. In this approach, multiple interacting continua are distinguished in the medium, each with different properties. A notable example is the dual porosity dual permeability (DPDP) model, which serves as a fundamental framework for simulating fractured oil and gas reservoirs [15, 17]. It comprehensively characterizes fluid flow dynamics by accounting for both flow within the fracture network and flow within the matrix. Early works of the multicontinuum approach include [17, 18]. Among the recent works, one can mention the multicontinuum homogenization method [19–21]. This homogenization method provides a flexible and rigorous mathematical technique for deriving multicontinuum models.

Another challenge of modeling filtration processes (especially for multicontinuum models) is time discretization. One can divide all time discretization methods into three main groups: implicit, explicit, and partially explicit. Implicit methods are widely used due to their unconditional stability [22]. However, these schemes come with a high computational cost for the numerical solution. On the other hand, explicit time discretization avoids the need for matrix inversion or costly iterative solvers, but it requires careful selection of time steps, constrained by the relationship between temporal and spatial discretization. Partially explicit (explicit-implicit) methods combine the best aspects of implicit and explicit approaches [23, 24]. In this temporal discretization, an implicit scheme handles fast processes, and an explicit scheme approximates slow processes. For example, an implicit scheme can be used for a higher permeable continuum, while an explicit scheme can be used for a lower permeable continuum.

Implicit methods are prevalent due to their stability, allowing the use of large steps in time for media with high permeability [22]. However, these schemes lead to a significant computational cost of the numerical solution and, in some cases, may not accurately track the dynamics of the solution. In contrast, explicit temporal discretization methods have low computational costs due to the few connections between degrees of freedom [22]. At the same time, explicit schemes can better track the high frequencies of the solution. However, the stability of such schemes depends on the size of the time step and the medium's permeability. Partially explicit (explicit-implicit) methods combine the best aspects of implicit and explicit approaches [23, 24]. In this temporal discretization, an implicit scheme handles fast processes, and an explicit scheme approximates slow processes. For example, an implicit scheme can be used for a highly permeable continuum,