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Characterizing Natural Convection, Energy, and Thermal Behavior in a Circular Cavity: Finite Element Analysis

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Abstract. This study improves understanding of the interaction between magnetic fields and natural convection in complex geometries, addressing a significant challenge in engineering design. By analyzing the effects of multidirectional magnetic fields on fluid flow and heat transfer, this study provides valuable insights into optimizing thermal management in practical applications, such as exhaust manifolds. This work is motivated by the need to enhance thermal efficiency and control in systems with irregular geometries, which are prevalent in advanced technological applications. Employing magnetohydrodynamic (MHD) principles, the flow and heat exchange dynamics are analyzed under real-world conditions, considering a cavity of length L with heated bottom walls and heated/cooled inner obstacles. The circular cavity contains heated rods with different radii. An inclined magnetic field, perpendicular to the fluid flow, is applied, while a no-slip condition is enforced at the walls. Finite Element Method simulations are conducted over a wide range of Rayleigh (Ra) and Hartmann (Ha) numbers with a fixed Prandtl number (Pr$=$6.2). The streamlines, isotherms, and two-dimensional (2D) plots are produced to show the effects of governing parameters. The present results are compared with the existing data and graphical results. The numerical results reveal that heat transfer is influenced not only by Rayleigh's number and magnetic field strength but also by the magnetic field's inclination.

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1 Introduction

The magnetic impacts in multidirectional and natural convection phenomena cause a complex scenario to design several engineering structures and originate a valued area for the engineers/scientists to see to the consequences while formulating systems with obstructions and modifying the governing designs to achieve desired conclusions. The collective consequences of natural convective and inclined MHD flow have been discussed and initiated a deep understanding of the interaction between buoyancy-driven flow and magnetic forces. The complex scenarios are noticed during the mechanical model formulation based on such phenomena. Besides, the circular or curvilinear geometrical shapes are difficult to deal with as the domain is irregular but possess several pivotal features in the practical world including lighting, low phase noise oscillators, optical communications photonics, power combiners in microwave, quantum information systems, resonances [1-3]. The stated variety of practical applications inspired the mechanical community to further numerically examine the area of mechanics by undertaking several pivotal characteristics and current research works are included for the readers.

Liao et al. [4] reported a mathematical formulation to examine a naturally convective persuaded by a thermal motivated stream under the impacts of the magnetic field inside a square-shaped enclosure.

The analysis is made for the variety of values of different prominent parameters which shows substantial magnetic impacts (for various angles) on streamlines and isotherms inside the cavity whereas the maximum streamline function (S-max) and the average Nusselt number (Nu-mean) are dropped with the magnetic field force. Numerical experimentations are made for a varied choice of Ha and Ra are carried out to examine the thermal pattern and an associated expression for heat transmission transition is anticipated to specify the shifting of the thermal pattern subjugated by the inclined magnetic field. Mohammadi and Nassab [5] studied a numerical aspect of optical thickness and inclined magnetic field impacts on the natural convective flow and thermal features inside a complex square geometry. The velocity and temperature distribution are analyzed through a multiple-time-relaxation lattice Boltzmann scheme. The found outcomes are described for higher Hartmann number, in an optically thin medium $(1 \geq \tau)$, the determined decrease of the heat transmission due to the magnetic influences 15%, despite the fact this is 61% and 66% at $\tau = 100$ and the non-radiative form, correspondingly.

Liao and Li [6] executed a numerical investigation for the thermal and naturally convective performance inside a square enclosure filled with nanofluids Al_2O_3 with inner obstruction and impacts of the magnetic field. The inclination of magnetic impacts on the critical values of the Hartmann number for the evolution of heat transmission approaches is examined and reported first time in the literature. It is concluded that the Nu