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Nonlocal Vibration Response of FG Saturated Porous Nanoplate Resting on Elastic Foundation Using Quasi-3D HSDT Theory

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Abstract. This study investigates the dynamic response of functionally graded saturated porous (FGSP) nanoplates resting on the elastic foundation. The porosities are assumed to vary gradually through the thickness of the plate, following three different patterns: uniform, non-uniform symmetric, and non-uniform asymmetric distributions. Biot's poroelasticity theory is employed to describe the stress-strain relation of functionally graded porous materials when in a liquid-saturated state. Additionally, the nanoscale effects of the structure are taken into account by incorporating Eringen's nonlocal elasticity theory. The equations of motion are formulated by applying Hamilton's principle, utilizing a quasi-three-dimensional higher-order shear deformation (quasi-3D HSDT) theory. This theory guarantees that the top and bottom surfaces of the nanoplate experience conditions free of transverse shear stress. The obtained results reveal that the free vibration and transient response of the FGSP nanoplate is significantly influenced by various factors, including the porosity coefficient and distribution patterns, geometrical parameters, elastic foundation stiffness, Skempton coefficient, and nonlocal parameters. The theoretical development as well as numerical solutions presented herein offer valuable insights and serve as a reference for nonlocal theories of FGSP nanoplates.

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1 Introduction

Functionally graded materials (FGMs) in general and particularly functionally graded porous materials (FGPMs) are employed not just in the production of macro structures but also in the fabrication of micro/nanostructures, such as structural components in aerospace and energy systems, heat exchangers and thermal barriers, electronics and semiconductors, medical scaffolds, and various other applications [1]. In particular, FG/FGP nanostructures are being exploited at an exponentially increasing rate [2]. Therefore, a thorough understanding of their mechanical behaviors is necessary. Studying and modeling nanoscale structures can be achieved through the use of experimental observations as one of the approaches. Due to the expensive nature of this approach, alternative methods, such as atomic modeling, density functional theory, and hybrid atomic continuum mechanics are employed. Continuum mechanics-based modeling has been demonstrated to be more cost-effective than other models. Because of the potential limitations of classical continuum mechanics in accurately modeling small-sized structures, the literature incorporates various size-dependent effects to address this concern. Taking into account small-scale effects, various continuum theories have been developed, including nonlocal theory [3–6], couple–stress theory [7–9], strain gradient theory [10–13], surface stress theory [14–16], and more. Eringen's nonlocal elasticity stands out as the most commonly utilized theory among these. In this context, the stresses at a point are influenced not solely by the strain at that specific location but also by the strains present within a certain surrounding region. This surrounding region is characterized by an internal characteristic length. In examining the mechanical characteristics of FGP structures, researchers utilize the poroelasticity theory pioneered by Biot [17]. Biot's theory encompasses two states: drained and undrained (saturated). In drained conditions, stress-strain relationships are articulated in accordance with the traditional theory of elasticity (Hooke's law). In addition to investigations into the influence of size effects on perfect nanobeams/tubes and nanoplates [18–22], many studies have been undertaken to highlight the porosity and size effect on FGP nanostructures in the drained state using nonlocal theory and different beam/plate theories, as detailed below.

Using the first-order shear deformation theory (FSDT), Karami et al. [23] studied the influence of various factors on wave frequency and phase velocity of clamped FGP nanoplates. Al-Furjan et al. [24] employed the Finite Element Method (FEM) and the Direct Quantum Method to investigate the nonlinear response of sandwich FGP nanoplates, utilizing various analytical models. In another study, Barati and Zenkour [2] used an analytical approach to examine the post-buckling behavior of FGP nanoplates. Employing shooting technique, Li et al. [25] analyzed the free vibration of a circular FGP micro/nanoplate. Doan et al. [26] used the FEM to study the free vibration of FGP