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Two Lattice Boltzmann Models for Three-Dimensional Coupled Burgers' Equations

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Abstract. Two lattice Boltzmann models are presented for the general three-dimensional coupled Burgers' equations with different diffusion coefficients. The coupled parts or all of the convection items are treated as the source terms, where the spatial gradient can be calculated by the distribution function. The three-dimensional coupled Burgers' equations can be exactly recovered from the lattice Boltzmann(LB) models through the direct Taylor expansion. Some numerical tests were performed to verify the accuracy and efficiency of the present models.

AMS subject classifications: 76M28, 65M12

Key words: Lattice Boltzmann model, three-dimensional coupled Burgers' equations, direct Taylor expansion, source term.

1 Introduction

As nonlinear partial differential equations (PDEs), three-dimensional coupled Burgers' equations can be used to describe some physical phenomena [1,2], such as wave propagation, sedimentation problems and many more. Some analytical methods [3–5] are used to derive the solutions of coupled Burgers' equations. Actually, many numerical methods are developed to solve the coupled Burgers' equations, for example, the

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finite difference method [6–8], variational iteration homotopy perturbation method [9], DQM [10], Laplace variational iteration method [11] and nodal Jacobi spectral collocation method [12], to name a few. In this paper, we use the mesoscopic lattice Boltzmann method(LBM) to solve the following three-dimensional coupled Burgers' equations in the computational domain D,

$$\frac{\partial u}{\partial t}+2\eta_1 u\frac{\partial u}{\partial x}+2\eta_1 v\frac{\partial u}{\partial y}+2\eta_1 w\frac{\partial u}{\partial z}=\alpha_1\Big(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\Big), \tag{1.1a}$$

$$\frac{\partial v}{\partial t}+2\eta_2 u\frac{\partial v}{\partial x}+2\eta_2 v\frac{\partial v}{\partial y}+2\eta_2 w\frac{\partial v}{\partial z}=\alpha_2\Big(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\Big), \tag{1.1b}$$

$$\frac{\partial w}{\partial t}+2\eta_3 u\frac{\partial w}{\partial x}+2\eta_3 v\frac{\partial w}{\partial y}+2\eta_3 w\frac{\partial w}{\partial z}=\alpha_3\Big(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\Big), \tag{1.1c}$$

where initial conditions are given as follows:

$$u(x,y,z,0)=\varphi_1(x,y,z)\subset D, \tag{1.2a}$$

$$v(x,y,z,0)=\varphi_2(x,y,z)\subset D, \tag{1.2b}$$

$$w(x,y,z,0)=\varphi_3(x,y,z)\subset D, \tag{1.2c}$$

and boundary conditions are

$$u(x,y,z,t)=\psi_1(x,y,z,t), \tag{1.3a}$$

$$v(x,y,z,t)=\psi_2(x,y,z,t), \tag{1.3b}$$

$$w(x,y,z,t)=\psi_3(x,y,z,t), \tag{1.3c}$$

where $(x,y,z)\subset\partial D, t>0$ and ∂D is the boundary of the given computational domain. $u(x,y,z,t)$, $v(x,y,z,t)$ and $w(x,y,z,t)$ are the components of velocity in three dimensions. φ_1, φ_2, φ_3, ψ_1,ψ_2,ψ_3 are the known functions, η_1, η_2 and η_3 are the parameters, α_1, α_2 and α_3 are the diffusion coefficients.

LBM is a numerical simulation method developed in recent decades for simulating various complex physical problems. LBM has the advantages of intrinsic parallelism, simple programme and easy to handle complex boundary conditions, which make the method successfully applied to study complex fluid flows [13–17] and solve various types of nonlinear PDEs [18–22]. As important PDEs, the coupled Burgers' equations are solved by LBM [23–26], where most of them only deal with one-dimensional or two-dimensional coupled Burgers equations. For three-dimensional coupled equation, Rong et al. [25] and Chen et al. [26] proposed two lattice Boltzmann models based on Cole-Hopf transformation for n-dimensional coupled Burgers' equations with potential symmetry conditions and the same diffusion coefficients. When the symmetry condition is not satisfied or the coupled Burgers' equations with different diffusion coefficients or $\eta_1\neq\eta_2\neq\eta_3$, which are rarely encountered for high dimensional coupled equations, the coupled Burgers' equations cannot be transformed into the diffusion equation through Cole-Hopf transformation in [25, 26]. To overcome these problems, based on the idea of source term, in this