Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **18**, No. 4, pp. 1087-1119

DOI: 10.4208/aamm.OA-2024-0193
August 2026

A New Composite Wall Function for Adverse Pressure Gradient Flows

Yiming Du, Mingze Ma, Jinrong Zhang, Hanqi Song and Chao Yan*

School of Aeronautic Science and Engineering, Beihang University, Beijing 100089, China

Received 29 August 2024; Accepted (in revised version) 17 April 2025

Abstract. This paper presents a composite wall function, which can be used for a wide range of the adverse pressure gradient (APG) and is derived from the composite wall-law provided in this study. The composite wall-law combines the characteristics of the log-law and the half-power law under the APG, and it degrades to the log-law under the zero pressure gradient (ZPG). A function relationship between the parameters in the composite wall-law and the dimensionless pressure gradient parameter p^+ is provided based on a series of DNS data for the APG. Finally, a generalized wall function incorporating both the viscous sublayer and the buffer layer is presented. The new wall function can accurately predict the mean velocity of both equilibrium and non-equilibrium flows under the APG. The new composite wall function is validated using three test cases: NACA0012, 2D channel with a bump, and turbulent separation bubble. The results demonstrate the accuracy of the new composite wall function under the APG flows.

AMS subject classifications: 65M10, 78A48

Key words: Adverse pressure gradient, wall function, turbulent boundary layers.

1 Introduction

In computational fluid dynamics, the use of wall function is often adopted to replace the no-slip boundary condition near solid walls, thereby reducing the grid requirements in the vicinity of the wall. The wall function relies on the wall-law to obtain accurate wall shear stress, to compute local turbulent eddy viscosity coefficient near the wall, and define boundary conditions. The application of wall function is based on the following fact: for incompressible flow, there exists a locally balanced boundary layer from the wall to the log-law region with similar solutions [1]. When using wall function, we can place the first grid layer in the log-law region, thereby eliminating the need to compute

*Corresponding author.
Emails: yimingdu@buaa.edu.cn (Y. Du), yanchao@buaa.edu.cn (C. Yan)

the flow field within and near the log-law region [2,3]. Furthermore, wall function are not only applicable to RANS (Reynolds-Averaged Navier-Stokes) solvers but can also be used in Large Eddy Simulation (LES) [4]. Currently, the wall function is widely used in commercial CFD software.

Accurate simulation of turbulence under APG is crucial for practical engineering applications. For example, in the case of an entire aircraft, complex flow phenomena such as secondary return flow at the fuselage and wing junctions, blunt body turbulence around the landing gear and inlet lips, and separation flows induced by pressure gradients due to high angles of attack or changes in the flow channel can significantly impact safety, maneuverability, and efficiency, and thus require accurate capture and prediction. Moreover, studies have shown that APG flows have a significant impact on transition prediction [5–8]. However, a widely accepted wall function for the inner layer that can describe a wide range of APG flows is not known. Compared to the extensive research on ZPG flows, the understanding of mean flow under APG is much more limited. Therefore, improving the accuracy of wall function for predicting mean flow under APG is both necessary and of significant importance.

Under the presence of a pressure gradient, many attempts to establish a generalized wall function have been investigate. Röber proposed a continuous form of the wall function with a pressure gradient by combining the analytical solution of the viscous sublayer and the van Driest damping function [9]. However, it shows significant deviation from experimental results when y^+ is large. Duprat et al. established a continuous wall function based on velocity scaling and the van Driest damping function, proposing another method for solving the continuous wall function with a pressure gradient [10]. When the wall distance is large, there is a significant difference between their results and experimental as well as computational results. Shih et al. proposed a new velocity distribution for the presence of a pressure gradient based on a new velocity scale that includes both viscous effects and pressure gradient effects [11]. They also performed a fitting for the buffer layer to establish a continuous wall function. Additionally, Li et al. [12] established a theoretical framework for the unified velocity law in hypersonic reactive turbulent boundary layers, while innovatively developing a wall function specifically optimized for compressible TBLs simulations.

The establishment of a generalized wall function requires a high-fidelity wall-law as its foundation. The wall-law for turbulent boundary layers (TBLs) under ZPG is given by the relationship between u^+ and y^+ [1], where u^+ and y^+ are obtained by solving simplified Reynolds-averaged equations [1,13–17]. The wall law originally used is the log-law [18]

$$u^+ = \frac{1}{\kappa}\log(y^+) + Bi. \tag{1.1}$$

Here κ and Bi are the von Kármán constant and the intercept respectively. And the superscript + indicates nondimensionalization using the wall friction velocity $u_\tau = \sqrt{\tau_w/\rho}$ and the viscous length ν/u_τ, where τ_w is the wall shear stress. The log-law can be derived through dimensional analysis [19]. In addition, Millikan also derived the log-law through