

# A Conservative Sharp Interface Method for Compressible Two-Medium Flows on Unstructured Triangular Meshes

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**Abstract.** A conservative sharp interface method based on the cut-cell approach is proposed for the simulation of compressible two-medium flows on unstructured triangular meshes. In this method, the governing equations for each medium are solved using a MUSCL-type finite volume scheme. The exact two-medium Riemann problem is solved at the moving interface, while the local Lax–Friedrichs flux is employed along the fixed cell edges. The interface is tracked by solving the advection equation for the level set function, and explicit interface reconstruction is performed at each time step within the cut-cell framework. To address the small cell problem and ensure global conservation, a novel cell-merging algorithm, specifically designed for unstructured triangular meshes, is introduced to ensure that interface cells coincide with the interface. In contrast to existing cut-cell methods used on Cartesian meshes, the complexity of the new algorithm in constructing cut cells is significantly reduced. Additionally, the adaptive mesh refinement technology is employed to efficiently simulate complex flow problems while reducing the computational cost. A series of numerical tests is conducted to demonstrate the conservativeness and accuracy of the proposed method.

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## 1 Introduction

Compressible two-medium flows play an important role in a wide range of scientific and engineering fields, including inertial confinement fusion [1], high-speed jet impacts [2], and astrophysical flows [3]. Over the past several decades, various numerical methods have been proposed and developed to simulate compressible two-medium flows. These methods can be broadly categorized into two main approaches: the diffuse interface method and the sharp interface method.

In the diffuse interface method, the material interface is treated as an artificial mixture region with some thickness, characterized by various parameters, such as mass fractions [4], volume fractions [5], or parameters associated with the equation of state [6]. Within the framework of diffuse interface models, the advection equations governing these parameters are coupled with the system's governing equations, in accordance with the established thermodynamic laws. This approach facilitates the straightforward extension of numerical methods originally designed for single-medium flow. However, two issues arise in the diffuse interface method. The first is the need to suppress oscillations near the interface, which arise from the unphysical mixing of different fluids [7], often leading to the use of non-conservative numerical schemes. The second issue is that the numerical diffusion tends to enlarge the interface region over time, particularly in long-term simulations, which results in a loss of accuracy in tracking the interface. To address this issue, ongoing research focuses on the development of the interface sharpening [8,9] and interface compression [10] techniques. Recent advancements in the diffuse interface method are summarized in the review paper by Saurel and Richard [11].

In the sharp interface method, the material interface is assumed to have zero thickness and is treated as a moving internal contact discontinuity. Over the past few decades, numerous methodologies have been developed, including the front-tracking method [12–14], the volume of fluid method [15], the moment of fluid method [16], the ghost fluid method (GFM), and the cut-cell method (CCM). In recent years, the GFM and the CCM have emerged as two mainstream approaches in the field of compressible two-medium flows. In the GFM, ghost fluids are assumed to exist on opposite sides of the interface, and each flow variable is individually updated using a single-fluid numerical scheme. A seminal contribution, referred to as the original ghost fluid method (OGFM) hereafter, was introduced by Fedkiw et al. [17]. In the OGFM, the velocity and pressure are directly obtained from another fluid, while the entropy is extrapolated to compute the density. Subsequently, several GFM-based methods have been developed, differing in how they extrapolate ghost fluid states and apply interface conditions. Examples include the modified ghost fluid method (MGFM) [18], the interface ghost fluid method (IGFM) [19], the real ghost fluid method (RGFM) [20], and the practical ghost fluid method (PGFM) [21], among others. A comprehensive analysis of these GFM-based methods was provided by Xu et al. [22]. However, all the aforementioned GFMs are non-conservative, primarily due to the discrepancies in fluxes in the vicinity of the interface. An in-depth analysis of accuracy and conservative errors can be found in [21]. The CCM was originally de-