

Analyzing Bending Problems of Plates on Elastic Foundations via Improved Element-Free Galerkin Method

Heng Cheng¹, Dongqiong Liang¹ and Fengbin Liu^{2,*}

¹ School of Applied Science, Taiyuan University of Science and Technology, Taiyuan, Shanxi 030024, China

² College of Aeronautics and Astronautics, Shanxi Key Laboratory of Material Strength and Structural Shock, Taiyuan University of Technology, Taiyuan, Shanxi 030024, China

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Abstract. In this study, by combining the orthonormal basis functions and the traditional moving least-squares (MLS) approximation, thus the new approximation function is established by using the improved moving least-squares (IMLS) approximation, and the corresponding formula derivation is given in Section 2. Afterwards, the equilibrium, geometrical and physical equations of bending problems of plates on elastic foundations are presented respectively, and the equivalent functional of such problems is established with the essential boundary condition imposed by using the penalty method, thus the calculation formula of numerical solution are derived using the improved element-free Galerkin (IEFG) method with IMLS approximation. In numerical examples, we verify the convergence of the IEFG method by increasing the number of nodes. In comparison with the EFG method, the IEFG method converges faster. Moreover, the smaller error and higher calculation speed are obtained by selecting the IEFG method for solving three numerical examples.

AMS subject classifications: 74K20, 74G15, 74S99

Key words: Meshless method, elastic foundation, thin plate, improved element-free Galerkin method.

1 Introduction

The plates on elastic foundations are widely used in engineering designing, including foundation design for buildings, bridges, roads and other projects. Through rational analysis and design, the safety and stability of the structure can be ensured and the damage caused by sink can be reduced. In order to prevent the deformation and sink

*Corresponding author.

Emails: liufengbin@tyut.edu.cn (F. Liu), chengheng@shu.edu.cn (H. Cheng), s202218110788@stu.tyust.edu.cn (D. Liang)

of foundation under external load, we need the corresponding mathematical model to calculate and prevent it. The common models include Winkler foundation model and Pasternak two-parameter elastic foundation model. The Winkler foundation model is a single parameter mathematical model, in which the pressure on any point of the foundation is directly proportional to the sink of that point, and the ratio of external force to subsidence displacement is called subgrade coefficient. However, this model ignores the shear stress in the foundation. The Pasternak two-parameter elastic foundation model simulates the characteristics of the elastic foundation more comprehensively by introducing two independent parameters. Based on the Winkler model, it also considers the influence of shear stress, leading to more accurate analysis.

From the 1960s to present, many numerical methods are studied to analyze the mechanics problems of plates on elastic foundations. Early in 1965, Cheung and Zienkiewicz [1] used the finite element method (FEM) to solve the bending problem of plate on elastic foundation. Ghosh et al. [2] studied the dynamics problem of cut-out plate on elastic foundation using the FEM. In addition, Puttonen and Varpasuo [3] proposed the boundary element method to analyze plate on elastic foundation. Different from the mesh-based methods, meshless method is the point-based approximation numerical method, which can be used for the solution of partial differential equation as well as mechanical problems. When discretizing the problem domain, meshless method only need some scattered points rather than dividing the grids. Therefore, it can save the cost of grid reconstruction when mesh distortion troublesome occur in which the FEM is used to solve large deformation problem. Currently, meshless method has gained attention by many scholars and evolved rapidly because of its advantage over the FEM.

In 2002, Sladek et al. [4] analyzed the simply supported and solidly supported slabs for two-parameter elastic foundations by using the meshless local boundary integral equation method. Rahbar-Ranji et al. [5] selected the EFG method to solve the buckling problem of variable thickness plates on elastic foundations. Bahmyari et al. [6] used the EFG method to solve the vibration of plates on Pasternak foundation. Al-Gahtani and Zenkour [7] used the quadratic radial basis function (RBF) method to obtain numerical solutions for different beams on elastic foundation. Al-Gahtani and Mukhtar [8] analyzed the free vibration of beam on elastic foundation by using the RBF meshless method. Al-Tholaia and Al-Gahtani [9] solved the large deflection of plates on nonlinear foundations by employing the RBF method, and the effectiveness of the method was verified. In order to obtain the bending solution of isotropic plate on elastic foundation efficiently, Chen and Cheng [10] proposed the complex variable reproducing kernel particle (RKP) method, and Wang et al. [11] established the complex variable EFG method. Selim and Liu [12] analyzed the impact effects of functionally gradient material (FGM) composite plates on elastic foundations using an element-free IMLS Ritz approach. Vu et al. [13] proposed a moving Kriging interpolation meshless method and used it to solve FGM plates on elastic foundations. Vu [14] studied a new numerical method by combining the advanced meshless method with the 3D hyperbolic shear deformation theory to analyze the mechanical behavior of porous plates on elastic foundations. Peng et al. [15]