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Multi-Resolution WENO Method with Adaptive Linear Weights using Subcell Strategy

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Abstract. This paper develops an adaptive-linear-weights, multi-resolution WENO method in the framework of subcell (AMWENO-SC). The features of AMWENO-SC present on that: first, it uses the subcell strategy; second, it only consists of two subreconstructions, where one is first order, the other is higher order and designed by using the CLS; third, it optimizes the smoothness indicator of first order subreconstruction by using the WLS-ENO; finally, it simplifies the adaptive linear weights by using the TENO approach. Compared with original AMWENO, our AMWENO-SC has better compactness, higher resolution and efficiency.

AMS subject classifications: 65M06, 76T10

Key words: AMWENO, subcell strategy, finite volume method, hyperbolic conservation laws.

1 Introduction

In order to solve the hyperbolic conservation laws well, there have been many popular numerical methods in the literature, such as ENO [7], WENO [21] and TENO [5]. These methods are efficient and robust to achieve the high order of accuracy in smooth region, and suppress the non-physical oscillation near discontinuity simultaneously.

Now we revisit the WENO method. Liu et al. developed the first WENO method [15], Jiang and Shu improved it to optimal fifth order of accuracy [10], Balsara and Shu extended it to higher order of accuracy [3], Hu and Shu proposed the genuine 2D WENO method on triangular meshes [8], and so on. This kind of WENO methods has to select the linear weights properly so that the high order of accuracy can be achieved in smooth region, however the linear weights may be negative or even non-existent. Over the last ten years, a kind of unequal-size WENO methods was developed for the free

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choice of linear weights. For example, Zhu and Qiu developed the USWENO method by using some unequal-size substencils, where one is used for high order subreconstruction, and the others are used for second order subreconstructions, it solves the stead-state problem well [32–34]; Balsara et al. proposed the similar USWENO method, however it can preserve the extremum and achieve the adaptive order [1,2]; Zhu and Shu built the MRWENO method, it realizes the increasingly higher order of accuracy by using a hierarchy of nested central spatial substencils [35,36]; Zhu et al. improved the MRWENO to AMWENO, which simplifies the framework by only consisting of two subreconstructions, and reduces the dissipation with the help of adaptive linear weights [12,13].

This paper employs the subcell strategy to improve the AMWENO, so that the resulted AMWENO-SC method has better compactness, higher resolution and efficiency. Here we revisit the existing methods using subcell strategy first. Wang et al. developed the spectral volume method(SV), which only uses the subcell averages in control volume for SV reconstruction [16, 24–27]; Pan et al. proposed the subcell finite volume method(SCFV) to reduce the dependence on grid quality [17,18]; Zhang et al. designed the SCFV gas-kinetic method [29–31]; Chen and Huang developed the XWENO-SC(X=DK and AO) method, which is the extension of XWENO in the framework of subcell [4]; and so on. In the further research, we find that, compared with DKWENO and AOWENO, the subcell strategy is more suitable to be applied on AMWENO method, because the shift from control volume to its subcell is simpler while the high order subreconstruction is constructed by using the constrained least squares [29]. We also find that, the smoothness indicator of first order subreconstruction introduced in AMWENO is too complicated, especially on unstructured meshes, so in our AMWENO-SC, the corresponding smoothness indicator is optimized by using the weighted-least-squares ENO technique developed by Liu and Jiao [14]. Furthermore, the adaptive linear weights introduced in AMWENO is also too complicated because the iteration is necessary, so in our AMWENO-SC, the adaptive linear weights are simplified by using the TENO technique developed by Fu et al. [5].

For the rest of this paper, we arrange it as follows. We first review the hyperbolic conservation laws and the corresponding AMWENO method on rectangular meshes in Section 2. Then in Section 3, we build the framework of AMWENO-SC method, and elaborate the design of high order subreconstruction, optimized smoothness indicator of first order subreconstruction and simplified adaptive linear weights. Subsequently in Section 4, we show the comparison between AMWENO and AMWENO-SC, which verifies the good numerical performance of AMWENO-SC. Finally we give the conclusion.

2 Review of fifth order AMWENO method

Consider two-dimensional hyperbolic conservation laws

$$U_t+F_x+G_y=0,\quad (x,y)\in\Omega=[a,b]\times[c,d], \tag{2.1}$$