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Machine Learning Based Optimization of Tube Geometry for Capillary Rise

Yanfang Xu[1], Zhenhua Chai[1,2,3], Fang Shan[1] and Xuhui Meng[1,2,*]

[1] *School of Mathematics and Statistics, Huazhong University of Science and Technology, Wuhan, Hubei 430074, China*
[2] *Institute of Interdisciplinary Research of Mathematics and Applied Science, Huazhong University of Science and Technology, Wuhan, Hubei 430074, China*
[3] *Hubei Key Laboratory of Engineering Modeling and Scientific Computing, Huazhong University of Science and Technology, Wuhan, Hubei 430074, China*

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Abstract. The phenomenon of capillary rise is commonly observed in both nature and engineering fields. There has been growing interest in the study of the geometry effect on the capillary rising. In this work, we investigate the effect of tube geometry on the capillary rise, and explore the optimal tube geometry that corresponds to the maximum equilibrium height. To this end, we present two machine learning approaches that are able to optimize the tube shape to obtain maximum equilibrium height given a material. Specifically, the first one is based on the Gaussian process regression (GPR) in a purely data-driven manner, and the second one utilizes deep neural networks (DNNs) which are capable of encoding the physical constraint, e.g., the governing equation for the capillary rise in a tube. We apply both methods to study the capillary rise in the uniform cylindrical, trigonometric-shaped and quadratic-shaped tubes, and then determine the optimal geometries that achieve the maximum equilibrium heights. Our results demonstrate that the both approaches can effectively optimize complex capillary channels corresponding to the equilibrium height. Further, GPR is computationally more efficient than DNNs, while the DNNs with physical constraints are more promising for solving problems in which the tube geometry is parameterized at high dimensions.

AMS subject classifications: 65L05, 65M99, 65Z05, 90C24

Key words: Capillary rise, Gaussian process regression, deep neural networks, optimal geometry, maximum equilibrium height.

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*Corresponding author.
Email: x2511530721@163.com (Y. Xu), xuhui_meng@hust.edu.cn (X. Meng)

1 Introduction

When the capillary tube contacts with the wetting liquid, the surface tension can overcome the influences of gravity and viscous forces, causing the liquid to rise along the tube until the meniscus reaches an equilibrium height under the balance of the capillary force and the gravity of the liquid column. As a common phenomenon in nature and daily life, the capillary rise has received increasing attention in the past decades [1–4], such as subsurface water flow in the soil, absorption or transport of water by plants, and the ink imbibition in fabrics. Also, there are many studies on the influences of the geometry, fluid properties and solid-liquid interaction on the capillary rising process [5–11]. However, the effect of tube geometry on the capillary rise process and how to design the tube to achieve the highest equilibrium height have not been fully understood.

The study of liquid flow in capillary tubes can be traced back to a century ago, Washburn [12] applied the Poiseuille's law to describe the fluid flow in a smooth circular tube, and suggested a diffusion correlation between the distance h and time t of the meniscus,

$$h=\left(\frac{\sigma R\cos\theta}{2\mu}t\right)^{0.5}, \tag{1.1}$$

where σ and μ denote the surface tension and the viscosity of the liquid, R is the tube radius and θ is the contact angle. This law holds under the condition $h\le h^{eq}$ (h^{eq} is the equilibrium height) [13] or when the tube is placed horizontally. For the fluid flow in a non-uniform tube, Reyssat et al. [14] studied the imbibition in the capillary channel with axial variation, and demonstrated that the kinetics of imbibition are closely related to the geometric details of the capillary channel. Staples et al. [15] focused on the capillary flow in a channel with a circular cross-section, whose diameter varies with a sinusoidal law. Additionally, some different channel shapes have also been considered, such as the stepped capillary [16], the V-shaped open grooves [17, 18], and assemblies of parallel cylinders [19]. Furthermore, some studies on the design of tube shape are also performed to accelerate the absorption of liquid. For instance, Shou et al. [20] developed a quantitative model to find the fastest liquid absorption in a two-segment heterogeneous porous structure. They also quantified the effects of local layer width and height on the capillary flow, and found that the capillary flow from wider to narrower cross-section is faster than that from the opposite flow direction in multi-section media [21]. Moreover, Figliuzzi et al. [22] utilized the optimal control theory to design optimized capillary in which the liquid can absorb faster than that in a uniform cylinder, and can rise 50% higher than the height in a uniform channel with the optimal radius.

In recent years, with the development of computer performance and parallel processing technology based on GPU and CPU, the deep learning [23] has shown some great advantages in treating high-dimensional data [24, 25] and predicting state variables of complex systems [26, 27]. Meanwhile, the deep learning-based inverse design (shape optimization) has also been applied in many fields, including nanophotonics [28], mechanical materials [29], and aerodynamics [30]. According to the universal approximation