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Local Interaction Simulation Approach for the Acoustic Wave Equation with Perfectly Matched Layer

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Abstract. The simulation of the acoustic wave equation is essential for a wide range of applications, but the complexity of the propagation medium presents significant challenges. These include addressing the infinite computational domain, which can be managed using absorbing boundary conditions, and handling interface conditions between different media. In this paper, we utilize the Local Interaction Simulation Approach (LISA) to simulate acoustic wave propagation, which is designed to accommodate complex interface geometries on a regular grid. In engineering applications, absorbing boundary conditions in LISA are typically implemented using the ALID method, which introduces a damping term that increases in the absorbing layer. Here, we propose integrating the Perfectly Matched Layer (PML) with LISA, a novel combination that has not been previously explored in the literature. We demonstrate that, despite the added complexity of PML due to its requirement for multiple auxiliary equations within the absorbing layer, it delivers superior performance and outperforms the ALID method in terms of effectiveness.

AMS subject classifications: 65M06

Key words: Acoustic wave equation, local interaction simulation approach (LISA), perfectly matched layer (PML), interface media.

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1 Introduction

Wave phenomena are prevalent, with acoustic waves being particularly significant due to their various applications. In this paper, we focus on the classical acoustic wave equation.

$$\frac{\partial^2 u}{\partial t^2}=c^2\Delta u=c^2\Big(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\Big), \tag{1.1}$$

where u is a scalar quantity representing the wave displacement, typically corresponding to variations in pressure, and $(c^2)^{-1}=\frac{\rho}{\mu}$ denotes the wave velocity, with ρ being the density and μ the bulk modulus of the propagation medium. When an external force generates a wave, a source term should be added to the right side of the dynamic equilibrium equation.

Wave propagation is an important topic with a rich history across various scientific and engineering disciplines, including seismology, electromagnetism, optics, and acoustics. The Fast Fourier Transform is an effective analytical method for addressing wave transmission, reflection, mode conversion, and nonlinear higher harmonics [25]. However, analytical approaches can only handle simple structures of infinite size, offering a basic understanding of wave propagation patterns. A more practical way to simulate wave propagation in complex media is through numerical methods. Classical numerical techniques include the Finite Element Method (FEM), Boundary Element Method (BEM), and Finite Difference Method (FDM).

When using classical numerical methods to simulate wave propagation, we encounter two significant challenges. The first issue is that the medium through which the wave propagates may be complex and inhomogeneous. Accurately simulating the wave propagation process requires the correct implementation of interface conditions; if these conditions are not well-defined, the numerical method may become unstable. Among existing numerical methods, the Finite Difference Method (FDM) is straightforward to derive, implement, and parallelize. However, its main drawback is accuracy, particularly when using smooth parameters at discontinuous interfaces [8]. To address this, Delsanto et al. proposed a powerful modeling technique called the Local Interaction Simulation Approach (LISA), based on explicit finite difference methods and a sharp interface model, for simulating wave propagation in isotropic, heterogeneous materials [6–8]. This approach enforces the continuity of displacements and stresses at the interface, enabling precise treatment of perfect interfaces between different materials, making LISA more accurate than pure FDM in scenarios involving discontinuous material properties. The accuracy of LISA was numerically justified in [31]. Additionally, LISA was later implemented in parallel using graphics processing units (GPUs) with Compute Unified Device Architecture (CUDA), demonstrating that GPU-based LISA outperformed FEM and reduced computation time from hours to minutes [20]. In recent years, numerous scholars have conducted extensive research related to this topic [3,4,17,18,23,26–28,30].

Another challenge in numerical simulations of acoustic waves is that they typically occur in infinite or semi-infinite domains. However, our primary focus is often on the