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Prediction of Transition Locations in Regions of the Minor Axis of Hypersonic Elliptic Cones Using the BiGlobal-e^N Method

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Abstract. Two-dimensional global (BiGlobal) stability in the region of the minor axis is investigated in the case of hypersonic elliptic cones with major-minor axis ratios of 2 : 1 and 3 : 1 at Mach number 6.0, and the BiGlobal-e^N method is proposed to predict the transition location of the boundary layer. Matrix-free BiGlobal stability analysis is used to find unstable modes, including the Y-mode and the Z-mode. The growth rates in the streamwise-frequency plane for these modes are obtained. The $N_{\max\_all}$ factor is proposed, which represents the maximum amplification factor that all BiGlobal unstable modes can reach. Using a comparison of the $N_{\max\_all}$ factor with the transition location measured in a wind tunnel experiment for the 2 : 1 elliptic cone, the transition prediction criterion is determined, i.e., $N_{tr} = 8.6$. In the transition position, the amplification factors of several modes reach a level close to 8.6, which implies that none of them has the absolute superiority sufficient to cause the transition itself. Finally, the BiGlobal-e^N method is employed to predict the transition location in the region of the minor axis of the 3 : 1 elliptical cone. It is found that a larger major-minor axis ratio leads to stronger instability and an earlier transition.

AMS subject classifications: 76E99, 76K05, 76F06

Key words: BiGlobal stability, transition, boundary layer, hypersonic elliptic cone, minor axis.

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1 Introduction

The prediction of the transition of a boundary layer in the region of the minor axis of a hypersonic elliptical cone is closely related to practical engineering problems, and it is of great significance for scientific developments. On the one hand, to develop a next-generation hypersonic vehicle, a transition prediction in the region of the minor axis of an elliptical cone is a key problem to be solved. On the other hand, transition prediction in the region of the minor axis depends on global stability analysis, at the leading edge of the study of fluid mechanics.

The stability of the boundary layer in the region of the minor axis of a hypersonic elliptic cone is quite complicated. Even at a zero angle of attack, the streamlines on both sides of the minor axis draw close to the axis to form streamwise vortices [1], as shown in Fig. 1. As Li et al. [1] pointed out, the boundary layer here changes significantly along both the wall-normal and circumferential directions, such that the conventional stability analysis that only takes wall-normal variation into consideration is not meaningful. It is necessary to use the global stability analysis, which takes both wall-normal and circumferential variation into consideration [2–4]. Consequently, if the e^N method predicts the transitions herein, the N factors should also be obtained through global stability analysis.

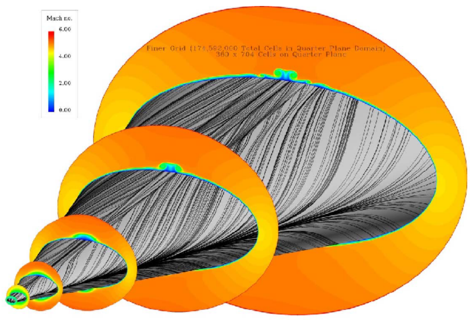

Figure 1: (color online) Basic flow of hypersonic elliptic cone HIFiRE-5 [1].

It took a historical process to understand the instability of the boundary layer and the transition in the region of the minor axis of the hypersonic elliptical cone. Basically, the historical process includes three stages. In the first stage, the streamwise vortices were not noticed. In the second stage, although the vortices were noticed, the global instability analysis method was not developed. In the last stage, the global stability instability analysis method was developed and adopted. The detailed introduction of the three stages is provided as below.

In earlier times, the streamwise vortices in the region of the minor axis were not noticed. Conventional linear stability theory (LST), which only considered wall-normal variation in the basic flow, was employed to study stability and predict transitions in the boundary layers of the elliptic cones, and the influence of crossflow was also considered.