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A Study of Fifth-Order WENO Reconstruction for Genuinely Two-Dimensional Convection-Pressure Flux Split Riemann Solver

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Abstract. Although the genuinely two-dimensional HLL-CPS solver holds the inherent multidimensionality property and capability of resolving contact discontinuities, the conventional low-order (second-order and below) reconstruction methods still limits its application in the two-dimensional complex flows involving shock waves and shear layers. A fifth-order reconstruction method is proposed for the genuinely two-dimensional HLL-CPS solver. The conserved variable vectors at the midpoints of interfaces are approximated by the fifth-order 1D WENO reconstruction. Meanwhile, variables at the corners are evaluated by a dimension-by-dimension reconstruction method consisting of a number of 1D fifth-order WENO sweeps. To avoid introducing spurious oscillations, each reconstruction is carried out in the corresponding local characteristic fields. Numerical results of several benchmark tests indicate the higher-order accuracy and the multidimensionality property of the proposed scheme. Compared with the 1D HLLE, HLLC and HLL-CPS schemes, the proposed high-order genuinely two-dimensional HLL-CPS solver provides higher resolution for contact discontinuities and presents better robustness against the shock anomalies.

AMS subject classifications: 65M10, 78A48

Key words: Genuinely two-dimensional HLL-CPS solver, WENO, fifth-order, dimension-by-dimension reconstruction.

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1 Introduction

Due to ease of implementation and satisfying physical properties, the Godunov-type numerical methods [26] based on approximated Riemann solver have become very popular in the simulations of compressible flows. The representatives of Godunov-type numerical schemes include the Roe's scheme [44], the AUSM-type schemes [35,41], the flux vector splitting (FVS) schemes [54,57], the HLL scheme [29], the HLLC scheme [56] and the Osher scheme [23]. Among them, the HLL Riemann solver has been widely used to solve different hyperbolic systems because of its simplicity and satisfying the entropy condition. However, the conventional HLL solver is incapable of resolving contact waves. In order to remove the limitation of the HLL solver, Mandal and Panwar [39] proposed the HLL-CPS scheme, in which flux vectors are split into convective and pressure components and the density difference of the dissipation term in the pressure flux term is replaced by the pressure difference to improve the resolution for contact waves. Furthermore, it has been proved to have the additional advantage of suppressing numerical shock instability [40].

Although the applicability of Riemann solvers mentioned above has been verified by a large number of numerical testes, some researchers indicated that the 1D Riemann solvers lose their efficiency in multidimensional problems since they can not pick up the flow information traveling along the transverse direction of a local interface. Therefore, many practitioners have attempted to construct genuinely two-dimensional Riemann solvers. Some of early works included the corner transport method by Collela [20], the introduction of multidimensionality into 1D solvers [45], the weighted average flux method proposed by Billet and Toro [16], Leveque's multidimensional wave propagation algorithm [36], Wendroff's HLLE solver for multidimensional hypersonic conservative laws [59], the multidimensional linearized Roe scheme by Fey [24] and Brio [19]. Compared with the traditional 1D solvers, the multidimensional solvers show some advantages in terms of accuracy. However, the mathematical complexity and high computational cost have limited their application in the simulations of complex flow problems.

Based on the idea of solving the two-dimensional Riemann problem at corners, Balsara [2] proposed the genuinely two-dimensional HLL Riemann solver. Unlike earlier multidimensional solvers, Balsara's solver is simple and easy to implement. Then, the genuinely two-dimensional HLL Riemann solver was extended to the unstructured mesh [7, 8, 14, 15, 18, 22]. Furthermore, Balsara et al. [4] developed a genuinely three-dimensional HLLE solver. Inspired by the work of Balsara, a series of effective genuinely two-dimensional HLL-type solvers have been constructed [17, 42, 48]. However, like 1D HLL solver, the genuinely multidimensional solvers derived from Balsara's method can not accurately capture shear waves. In order to overcome this defect, Balsara [3] constructed a genuinely two-dimensional HLLC solver, but this model is much complex and computationally expensive. Facing the challenge to construct an efficient two-dimensional solver with the capability of capturing contact discontinuities, Mandal and Sharma [40] introduced the convective-pressure splitting (CPS) strategy into the Bal-