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Semi-Discrete Predictor-Multicorrector FEM Algorithms for the 2D/3D Unsteady Incompressible Micropolar Fluid Equations

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Abstract. In this paper, the first-order and second-order semi-discrete predictor-multicorrector (PMC) algorithms to solve the 2D/3D unsteady incompressible micropolar fluid equations (IMNSE) are proposed. In the algorithms, the first-order and second-order BDF formulas are adopted to approximate the time derivative terms. At each time step, two elliptical sub-problems with Dirichlet conditions are solved at the prediction step, the strategy of projection about linear momentum equation with additional viscosity term and the elliptical sub-problems about angular momentum are solved at the multicorrection step. Furthermore, the unconditional stability and error estimates of the first-order scheme are proved theoretically. Numerical experiments are carried out to show the effectiveness of the algorithms.

AMS subject classifications: 76M10, 65N12, 65N30

Key words: Micropolar fluid equations, predictor-multicorrector algorithm, finite element method, error estimates.

1 Introduction

The micropolar fluid models can be used to describe the evolution of incompressible fluid in which some microcosmic properties of the material, such as translational and rotational degrees of particles, are considered. The first micropolar fluid model was introduced by Eringen in 1966 in [1], then, the thermomicropolar fluid model was introduced by him in 1972 in [2]. Later, the magneto-micropolar fluid model was introduced by Ahmadi and Shahinpoor in 1974 in [3]. The incompressible micropolar fluid problems have been widely used in modern industry, biology, and other fields, such as bearing lubrication, blood, and some other applications [4, 5].

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There are many mathematical studies on the existence and uniqueness of the solutions of IMNSE. Such as, the existence and uniqueness of weak and periodic solutions of unsteady IMNSE are analyzed in [6]. The existence and uniqueness of strong solutions and the existence of some local solutions and global solutions are also analyzed in [7]. Also, the finite element methods for solving IMNSE have been widely used. Among them, Ortega and Rojas applied the fully discrete and penalized finite element algorithms to solve the unsteady IMNSE in [8], and proved the optimal error estimates of linear velocity, angular velocity, and pressure. Jiang and Yang proposed the first-order and second-order projection algorithms in [9], in these methods, the corresponding problem is decoupled into two linear elliptical problems and one Poisson problem at each time step, and the stability and optimal error estimates of the first-order scheme were analyzed. Maimaiti and Liu proposed the first-order and second-order modified pressure-correction algorithms in [10] to solve the unsteady IMNSE, in which, the stability and error estimates were analyzed, but the nonlinear terms were not considered. After this, the corresponding algorithms have been used for inhomogeneous and curved boundary conditions. Xing and Liu proposed the three iteration methods (Stokes, Newton and Oseen iterative methods) to solve the steady IMNSE in [11], in which, the characteristics of the three methods were analyzed and compared, and the stability and error estimates of Oseen iteration method were derived. Liu et al. proposed four stabilization algorithms (penalty, regular, multiscale enrichment, and local Gauss integration methods) in [12], which were used to solve the steady IMNSE, the error estimates were analyzed and the characteristics of the four methods were compared.

One of the difficulties in numerically solving IMNSE is that when the nonlinear effect is strong, means that the viscous effect is weaker. The above difficulty can be optimized by the predictor-corrector (PC) algorithm. PC algorithms have been applied to the Navier-Stokes equations, such as Blasco and Codina proposed the PMC algorithm to solve the Navier-Stokes equations in [13], but the convergence about the algorithm was not analyzed in the paper. Next, Codina and Folch proposed a PC algorithm based on the orthogonal subscale stabilization to solve the incompressible Navier-Stokes equations in [14], which could not only deal with the flow field dominated by convection but also allow the use of the same order linear velocity and pressure interpolation. The above algorithms have been widely used, for more information about these algorithms, please refer to references [15–17].

In the case of strong nonlinearity, the convection effect is enhanced. In order to maintain the computational stability well, PMC algorithms are adopted in the paper. In addition, BDF1 and BDF2 formulas are used to approximate the time derivative terms to modify the time error estimates. Last, the unconditional stability and error estimates about the first-order scheme are analyzed. The main conclusions are Theorems 3.1-3.3.

The paper is organized as follows. In Section 2, the problems and some basic knowledge are given. In Section 3, the time semi-discrete format of the first-order and second-order schemes are proposed, also, the stability analysis and error estimation of the first-order schemes are proved. In Section 4, some numerical experiments are presented. Fi-