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Exploring Acoustic and Sound Wave Propagation Simulations: A Novel Time Advancement Method for Homogeneous and Heterogeneous Media

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Abstract. In this article, we have developed a new time-marching method to simulate sound and soliton propagation in homogeneous and heterogeneous media. The proposed time integration scheme can numerically preserve physical dispersion over a wide wavenumber range and conserve energy while solving wave propagation problems. Here, the Fourier stability analysis has been used to assess the numerical properties of the developed method. The proposed numerical method's dispersion and dissipation properties have also been compared with the classical fourth-order Runge-Kutta (RK4) method. Stability property contours for the newly proposed method display that the maximum allowable time step is at least five times higher than the RK4 method. The Fourier stability analysis also explains the dispersion error associated with the used spatio-temporal discretization schemes. It is observed that the dispersion error is significantly small for the proposed time integration schemes compared with the RK4 method. The proposed methods simulate sound propagation problems with fewer computational resources that otherwise demand high computational costs. The efficacy of the proposed time integration methods has been demonstrated by solving benchmark sound wave propagation problems. Moreover, to test the developed method's efficiency and robustness, we have performed simulations of the sound wave propagation in a layered media, corner-edge model, and damped sine-Gordon equation.

AMS subject classifications: 65Mxx, 76Qxx, 76B15

Key words: Time stepping, acoustic wave propagation, sound wave propagation, compact schemes, energy conservation, sine-Gordon equation.

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1 Introduction

Engineering problems involving sound wave propagation, reflection, diffraction, and superposition, are computationally intensive and challenging [36, 43]. Acoustic disturbances are pressure fluctuations superimposed on the background atmospheric pressure field. These pressure fluctuations' amplitude is usually a few orders smaller than the background pressure field. The viscous nature of the air does not alter the amplitudes of these fluctuations when the acoustic wave propagation is considered over a small distance [24]. The propagation of acoustic waves inside a one-dimensional (1D) domain displays a non-dissipative and non-dispersive nature. However, most of the traditional discretization schemes show a dispersive and dissipative nature across a wide wavenumber range. Dispersion and dissipation errors are usually high for the high wavenumber components when the computations are performed over large CFL (Courant-Friedrichs-Lewy) numbers. Thus, to keep dispersion and dissipation errors minimal, one is forced to compute on a highly refined grid with a significantly smaller time-step. Most of the schemes become unstable at higher CFL numbers. Thus, we have to use very small time-steps while performing computational aeroacoustic (CAA) simulations, which increases the computational cost significantly.

Simulations of some of the engineering problems, such as analysis of material defects using sound waves or sound wave propagation in seawater, involve heterogeneous mediums. Such problems pose further challenges as the medium properties change spatially, causing spatial variation in the speed of sound. One observes sound wave reflections and diffraction due to spatial gradients or discontinuities present in the medium properties. So far, we have listed computational challenges associated with linear acoustics problems where the frequency of sound waves in the domain is the same as the excitation or driving frequency. If the amplitude of the sound waves is sufficiently large, then the nonlinear effects also become important. If the frequency of the sound waves is sufficiently large, one must also consider the attenuation of sound waves due to the dissipative nature of the medium. Thus the used discretization scheme should have the ability to capture complex physical phenomena while solving the governing partial differential equations. To address these challenging issues, here we have proposed and used a new time integration scheme (EDP_1) along with the second-order centered (CD2) and sixth-order compact (C6) [26] spatial discretization schemes. Authors in [9] proposed the unconditionally positive definite finite difference (UPFD) method, which has been thoroughly validated [2] for the advection-diffusion-reaction systems. Authors in [34] developed unconditionally stable methods by utilizing the combination of the hopscotch spatial structure and leapfrog time integration to solve the time-dependent diffusion equation. Moreover, by considering odd-even hopscotch structure, authors in [35, 40] developed novel numerical methods to solve time-dependent diffusion equation. Compact difference schemes on uniform grid spacing have recently gained significant popularity for simulating flow and wave propagation problems on collocated and staggered grids [1, 19]. Regions with large gradients necessitate fine grid spacing, whereas regions