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A Boundary Mapped Collocation Method for the Analysis of the Arbitrarily Shaped Plates

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Abstract. An innovative meshless method is proposed in this paper for the bending problem of arbitrary Kirchhoff plates subjected to external force with various shapes and different boundary conditions. Without using a numerical integral, the deflection of the thin plate is approximated by using the boundary mapped collocation approach. Moreover, the computational domain discretization is just dependent on discretized nodes on the axis, while tensor product nodes have been mapped in the domain automatically. Hence, in the boundary mapped collocation implementation, the approximation functions are derived by employing the one-dimensional moving least squares technique for two-dimensional and higher-dimensional problems. Further, the virtual boundary technique is introduced to enforce the boundary conditions in the proposed method. Additionally, four numerical experiments are presented to illustrate the excellent convergence and high precision of the proposed approach.

AMS subject classifications: 35-04, 65N35, 68-04

Key words: Boundary mapped collocation method, moving least squares, Kirchhoff plate, meshless method.

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1 Introduction

Exploring the constructional mechanical behavior via solving various partial differential equations for engineering problems has always been one of the most challenging tasks for engineers and scientists [1]. However, it is difficult to derive analytic solutions for complex partial differential equations (PDEs). Hence, several numerical algorithms [2–4] have been proposed for solving the various PDEs [5–7]. Among these are the grid-based methods, such as the finite element method (FEM), the finite difference method (FDM), and the boundary element method (BEM) are widely used methods of numerical simulation to practice engineering. In Kirchhoff plate bending theory, flexural functions satisfy the fourth-order PDE of the plate. Since the high-order properties of the governing equation, the grid-based approaches all have faced numerous challenges e.g., the computational cost of the FEM method is high for solving high dimensional problems, the fictitious boundary technique is introduced to eliminate singularity of fundamental solutions in method of fundamental solution [8, 9]. Also, the fourth-order singularity of the kernel functions requires a complicated boundary integration technique by employing the traditional BEM. Therefore, it is of great academic interest to develop a computationally efficient and accurate numerical approach.

Remarkably, as a new numerical technique that obtains approximate solutions for PDEs by getting rid of mesh dependence, meshless approaches have recently received increasing attention in the fields of computational mechanics and computational mathematics. Since the beneficial properties such as simplifying pre-processing, high precision, and the possibility of solving partial equations without resorting to any background grids in the computation domain, meshless approaches have become an effective alternative to the conventional grid-based technique [10].

In this regard, some meshless methods have been directly adopted for solving numerous practical engineering problems [11, 12], the diffuse element method (DEM) [13], the element-free Galerkin (EFG) method [14], the reproducing kernel particle method (RKPM) [15], the h-p clouds methods [16], the meshless local Petrov-Galerkin (MLPG) method [17], the finite point method (FPM) [18], the local Petrov-Galerkin approach with moving Kriging interpolation [19–21], the element-free kp-Rize method [22], the meshless with boundary integral equation methods [23–25], the interpolating element-free Galerkin method [26], the complex variable meshless method [27], the Burton–Millertype-type singular boundary method [28], the localized Chebyshev collocation method [29], the localized method of fundamental solutions [30] and the boundary collocation method [31]. In this regard, meshless techniques are particularly adapted for the analysis of the plate and shell structure bending problems. Krysl and Belytschko in [32, 33] approximated the deflection by employing the EFGM. Sadamoto and Tanaka et al. in [34] introduced the RKPM for studying plate and membrane problems. Leitão in [35] proposed radial basis functions (RBFs) to analyze the Kirchhoff plate problems. Sadamoto and Ozdemir et al. in [35] solved buckling problems for cylindrical shells employing the reproducing kernel (RK) meshfree method. Long and Atluri in [36] introduced the MLPG