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Lattice Boltzmann Method for 2D Tidal Flow: Application to the Nador Lagoon

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Abstract. The lattice Boltzmann method is a relatively modern method. It has been widely applied to industrial flows (for small scales) governed by the Navier Stokes equations, but little applied to large-scale flows, such as geophysical flows. In this paper, we then attempt to apply the lattice Boltzmann method to coastal flows governed by the Saint-Venant equations (also called Shallow water equations) via an application to the real case of the Nador lagoon (Moroccan Mediterranean coast).
The establishment of the equations of the model and their discretization is given here. The introduction of turbulence by the large eddy simulations was also presented. The verification of the developed code concerns two test cases in hydraulic engineering. The first verification test deals with complex flow in a 180° bent channel. For this case, the model has faithfully reproduced all the characteristics of this flow (pressure/depression zone and the recirculation zone after the bend). Similarly, the results of this test case have been successfully compared to measurements available in the literature. The second verification test analyzes the flow around a spur-dike. Finally, the developed code has been adapted for its application to the real case of the Nador lagoon whose flow is generated by the superposition of the tidal waves M_2 and S_2. The model reproduced with good accuracy the dynamics of the flow and more particularly the two vortices observed on both sides of the lagoon inlet. The founding results are successfully compared with the results of previous studies published for the lagoon.

AMS subject classifications: 65M10, 78A48

Key words: Lattice Boltzmann method, multiple relaxation time, shallow water equation, Nador lagoon, tidal wave.

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1 Introduction

The world's most vulnerable and fragile ecosystems are coastal lagoons. These areas are habitats for various species and offer crucial environmental services, including fisheries. These lagoons are physical systems where salinity and hydrodynamics vary significantly, and their value in preserving biodiversity is widely acknowledged. A coastal lagoon is shallow inland water located alongside the coast, isolated from the sea but connected by a narrow channel, often called the inlet. The inlets are vital in developing barrier islands and preserving the lagoon's ecological system for numerous distinct causes such as increasing aquaculture, reducing water levels to prevent flooding on urban properties, and increasing oxygen levels [1]. Nevertheless, these inlets may open or close for purely natural or artificial reasons, drastically affecting the features of the lagoon [2].

Moreover, several types of animals and plants can be discovered in lagoons. Due to their richness, coastal lagoons are desirable locations for humans to engage in a variety of important activities. Unfortunately, despite their importance in preserving the ecosystem, coastal lagoons are subject to multiple pressures from human intervention which, often, generates catastrophic consequences (chemical contamination, traffic of oil tankers, and so on) and doubtlessly impact the balance of this environment. Thus, preliminary research must be done, in order to minimize the impacts of these consequences. Additionally, an understanding of the mechanisms governing the structure of lagoon ecosystems is required for this research. In light of this, computer software and numerical models have been developed through the years to serve as critical tools for researching and understanding the dynamics of water flows in coastal lagoons.

One of the most important lagoons on the coastlines of Morocco is the lagoon of Nador. This lagoon, recently, has been the subject of numerous studies and investigations on flora, fauna, water dynamics and human interventions [3]. Also, in the literature, there are some examples of modeling sediment transport and water circulation, among others; Benkhaldoun et al. [4] used an adaptive finite volume non-homogeneous Riemann solver (NHRS) for solving the sediment transport in this lagoon. Chaabelasri et al. [5] have simulated the movement of passive pollutants via tidal and river circulation within the lagoon using a $2D$ finite volume model that combines the shallow water equations (SWE) and the transport equation. Moreover, Jeyar et al. [6] have used a $2D$ finite volume model based on the SWE to simulate the effect of wind and tidal waves on the hydrodynamics circulation in the Nador lagoon. Lately, Jeyar et al. [7] describe the particle residence time in Nador lagoon using an Eulerian advection-diffusion technique. Thus, according to the literature review, the authors mostly used the finite volume method (FVM) and finite element method (FEM) to solve hydrodynamics problems. However, and to the best of the authors' knowledge, there are no research studies dealing with the numerical simulation of tidal waves in the Nador lagoon using the lattice Boltzmann method (LBM).

In this paper, we propose the numerical simulation of the coastal lagoon using the LBM. For the past twenty years, the LBM has been regarded as a crucial replacement for traditional computational fluid dynamic (CFD) method for simulating fluid dynam-