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The Random Feature Method for Elliptic Eigenvalue Problems

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Abstract. Solving eigenvalue problems is an important subject in scientific computing. Classical numerical methods, such as finite element methods and spectral methods, have been developed with great success in applications. However, these methods struggle in the case of complex geometries. The recently developed random feature method has demonstrated its superiority in solving partial differential equations (PDEs), especially for problems with complex geometries. In this work, we develop random feature methods for solving elliptic eigenvalue problems. Our contributions include (1) using tailored separation-of-variables random feature functions to approximate eigenfunctions, (2) employing collocation points in the strong formulation or quadrature scheme (exact integration) in the weak formulation to handle the PDEs and boundary conditions, and (3) solving the generalized eigenvalue problem in which the number of conditions equals the number of unknowns. Through a series of examples from one dimension to three dimensions, we demonstrate the high accuracy and robustness of the proposed methods with respect to geometric complexity. We have made our source code publicly available at `https://github.com/lingyun-2024/The-code-of-RFM-EEP.git`.

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Key words: Eigenvalue problems, elliptic eigenvalue problems, machine learning, random feature method, collocation method.

1 Introduction

In scientific and engineering applications such as structural dynamics [2], quantum che-

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mistry [6], electrical networks [7], magnetohydrodynamics [10], and control theory [16], numerical solution of eigenvalue problems plays a crucial role. Various methods have been developed to solve these problems, including finite element methods [8, 15, 17], spectral methods [4, 12, 17], and spectral-element methods [1, 18]. These methods have been successful in a range of applications and have been sufficiently analyzed in theory. Despite their success, traditional methods encounter challenges when dealing with complex geometries and high-dimensional problems. With the rapid development of deep learning in recent years, deep neural network-based methods have shown remarkable advancements in solving PDEs and eigenvalue problems [13, 14], particularly in high-dimensional problems.

The purpose of our paper is to develop random feature methods (RFMs) for solving elliptic eigenvalue problems. RFM is a natural connection between traditional and neural network-based algorithms, which has had great success in solving PDEs, especially in handling complex geometries [9]. We extend this method to solve elliptic eigenvalue problems, as traditional methods such as finite element methods, spectral methods, and spectral-element methods encounter difficulties in solving elliptic eigenvalue problems involving complex geometries.

For solving PDE problems, RFM transforms the original problems into least-squares approximation problems. However, for solving eigenvalue problems, RFMs convert the original problems into generalized eigenvalue problems. Firstly, we utilize random feature functions to approximate eigenfunctions. Then, we employ collocation points in the strong formulation or a quadrature scheme (exact integration) in the weak formulation to handle the PDEs and boundary conditions. These processes result in a generalized eigenvalue problem, where the number of conditions equals the number of unknowns. Finally, we solve the generalized eigenvalue problem to obtain numerical results.

The structure of the paper is as follows. In Section 2, we first introduce elliptic eigenvalue problems. Then we present the RFMs for elliptic eigenvalue problems: including the construction of separation-of-variables random feature functions, the strategies (the strong formulation and the weak formulation) for handling both the PDEs and boundary conditions, and the treatment of generalized eigenvalue problem. The strong formulation is discussed in Section 3, while Section 4 focuses on the weak formulation. In Section 5, we present a range of numerical experiments from one dimension to three dimensions, which demonstrate the high accuracy and robustness of our methods with respect to geometric complexity. Section 6 presents several conclusions.

2 The random feature method for elliptic eigenvalue problems

In this section, we introduce elliptic eigenvalue problems in Section 2.1. Next, we demonstrate the construction of random feature functions in Section 2.2, which are utilized to approximate the eigenfunctions.