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Physics-Informed Deep Learning Method for Surrogate Modeling in Elastic Mechanics

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Abstract. In this paper, we utilize Physics-Informed Neural Networks (PINNs) without any labeled data to solve the fourth-order partial differential equation governing the bending of thin plates. We meticulously formulate and apply our framework to the bending problem of elastic thin plates. Our findings indicate that defining each solution variable through an independent network, meaning without shared network parameters, yields superior performance compared to a single-network model with multiple outputs. During the training process, we discovered that more accurate results were achieved by adjusting the network architecture to strictly satisfy the boundary conditions, rather than incorporating them as part of the loss function. Remarkably, the PINNs are capable of obtaining relatively good results without the use of any labeled data during training, irrespective of whether soft or hard constraints are applied. In the end, the neural network's predictions for plate deflection, stress, strain, bending, and shear force all have errors less than 5% compared to the analytical solutions. In the algorithm implementation phase, we leverage the Python library DeepXDE, which facilitates the training of PINNs by providing an efficient and expedited process.

AMS subject classifications: 65M10, 78A48

Key words: PINNs, elastic thin plates, independent network without labeled data, DeepXDE.

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1 Introduction

With the rapid development of artificial intelligence (AI) and supercomputers, deep learning baseed on deep neural networks (DNNs) has been successfully applied in many research fields, including computer vision [1], natural language processing [2], self-driving cars [3], biological science [4] and generative modeling [5]. Despite the remarkable success in these and related fields, deep learning has yet to be widely applied in scientific computing. A common difficulty with the state-of-the-art machine learning (ML) techniques has been due to the extremely complex and expensive data acquisition in a vast majority of complex engineering/scientific systems. This threatens the feasibility and reliability of machine learning and renders conclusion and decision making a formidable task, if not impossible. A promising remedy in dealing with such problems is the prior physical knowledge, typically accessible in forms of Ordinary/ Partial Differential Equations (i.e., ODEs/PDEs), which plays the role of a regularization agent to consolidate the admissibility of the solution, despite access to limited data.

Raissi et al. [6] proposed a physics-informed neural network (PINN) to solve forward and inverse problems of partial differential equations (PEDs) with a moderate amount of labeled data. The training of PINNs is performed with a cost function that, in addition to data, includes the PDEs, initial conditions (ICs) and boundary conditions (BCs), which greatly reduces the dependence on training datasets without sacrificing generalization capability. This framework has been applied and promoted in many fields.

In solid mechanics, Guo et al. [7] utilized a PINNs with a single output to approximate the deflection of Kirchhoff plates. They demonstrated that this method is applicable for bending analysis of Kirchhoff plates with various geometric shapes. Then, Haghighat et al. [8] and Roy et al. [9] proposeed a physics-informed multi-network model that results in more accurate representation of the field variable in linear elasticity. Subsequently, the method of the energy equations as physical information into the neural network [10–12] has been used to solve the bending and buckling problems of Kirchhoff plates. This method directly solves the energy functional with second-order partial derivatives, with the Newman boundary conditions already included, thus only the Dirichlet boundary conditions need to be considered. As a result, it offers faster solution speeds, with no significant difference in accuracy, and in some cases, even higher precision. Roy et al. [13, 14] employed a data-driven deep learning framework based on PINNs theory to address von Mises plasticity and the non-associative Drucker–Prager elastoplastic constitutive model. They also utilized transfer learning to accelerate model training speed.

In fluid mechanics, Rose et al. [15] proposed two subgrid-scale (SGS) models based on deep learning, namely the Tensor Basis Neural Network (TBNN) and the Galilean Invariant Neural Network (GINN), for turbulence modeling in large eddy simulations (LES). Mao et al. [16] proposed PINNs to solve problems related to high-speed aerodynamics, specifically focusing on the Euler equations.

In geomechanics, Zhang et al. [17] proposed a hybrid deep learning model integrating both data-driven and physics-based strategies to decrease calculation costs and eliminate