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A Modified Regularized Lattice Boltzmann Method for Coupled Thermal-Solutal Problems of Non-Newtonian Fluids in Heterogeneously Porous Media

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Abstract. In this study, we present a modified regularized lattice Boltzmann method (RLBM) designed to simulate double-diffusive convection of non-Newtonian fluids within heterogeneously porous media. The modification involves the incorporation of correction terms related to shear rate and heat (or concentration) flux into the evolution equations of the RLBM for hydrodynamic equations and convection-diffusion equation, respectively. This allows shear-dependent viscosities and diffusion coefficients to be controlled by additional parameters, enabling the relaxation coefficients in the collision process to remain fixed at optimal values. Through multi-scale Chapman-Enskog analysis, the modified RLBM accurately recovers the governing equations for double-diffusive convection of non-Newtonian fluids within porous media at the representative elementary volume scale. The validity of the method is demonstrated by simulating double-diffusive natural convection in a two-dimensional porous cavity filled with power-law and viscoplastic fluids, with the numerical results showing strong agreement with established data from prior studies. Additionally, the influences of key dimensionless parameters, such as porosity, Bingham number, and power-law index, are thoroughly examined across a range of values.

AMS subject classifications: 65Z05, 76W05

Key words: Lattice Boltzmann model, double-diffusive convection, non-Newtonian fluid, porous media.

1 Introduction

The research on fluid flow coupled with heat and mass transfer within porous media has garnered increasing attention due to the complex challenges arising from the inter-

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play between heat and mass transfer [1, 2]. This area of research has been the subject of extensive numerical and experimental investigations, owing to its wide-ranging applications in both natural processes and industrial fields [3], such as biofluidics [4], petroleum engineering [5], chemical engineering, injection molding, groundwater pollutant migration, and hydraulic fracturing [6]. Many fluids involved in these applications, as well as their interactions with porous structures, exhibit non-Newtonian behavior [7]. As a result, the study of non-Newtonian fluid flow through porous media has attracted considerable interest from researchers in various scientific and engineering disciplines [8]. In response, numerous models have been developed in the literature to describe different types of non-Newtonian fluids under various flow conditions, including power-law fluids, viscoelastic fluids and viscoplastic fluids.

Recently, the study of double-diffusive convection (DDC) of non-Newtonian fluids in porous media has received considerable attention due to its critical applications across various fields. The nonlinear stress relationships with heat, and mass transfer significantly increase the complexity of DDC analysis, presenting substantial challenges for numerical simulations. The lattice Boltzmann method (LBM) has emerged as an effective mesoscopic numerical technique for modeling a broad range of complex fluid flow phenomena [9–15]. Its advantages, including a parallelizable algorithm, simplicity, and ease of implementation, have led to its widespread use in addressing fluid flow and thermal-solutal problems within porous media [16–18]. When utilizing LBM to study fluid flow, heat, and mass transfer in porous media, researchers typically employ one of two general approaches: the pore-scale method or the representative elementary volume (REV) scale approach. The pore-scale method, while highly detailed, is computationally intensive and requires substantial resources. In contrast, the REV model is more computationally efficient, as it relies on average transport properties, making it easier to implement. Recent advancements have led to the development of several LBM models based on the double-distribution-function (DDF) method at REV scale to address heat and mass transfer challenges in porous media. For instance, He et al. [19] investigated fluid flow, heat, and mass transfer in heterogeneous porous media with temperature-dependent viscosity using the LBM at REV scale. Chen et al. [20] developed a novel LBM approach to simulate natural convection in large Prandtl number fluids within porous media, demonstrating that the heat transfer characteristics and correlations differ markedly from those in small Prandtl number fluids. Kefayati et al. [21, 22] conducted numerical studies on DDC in non-Newtonian power-law fluids within an inclined porous cavity, accounting for the Soret and Dufour effects. Their work provided an in-depth analysis of entropy generation, fluid flow, and heat and mass transfer using the finite difference LBM. Building on this, Kefayati et al. [23] developed a generalized LBM for thermal incompressible non-Newtonian fluids in porous media and performed numerical studies on the natural convection of Bingham and power-law fluids within a porous cavity. Furthermore, they [24] simulated DDC in a porous cavity filled with viscoplastic fluids using LBM and recently proposed a mesoscopic model for DDC problems in power-law fluids within porous media, presenting macroscopic equations for various REV models [25].