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Mathematical Study of Fractional Magnetohydrodynamic Blood Flow Nanofluid in Activation of Thermal Radiation with Wright Function

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Abstract. A study of magnetohydrodynamics model of blood flow is made with single walls carbon nanotubes, copper (Cu), tin (TiO_2), and alumina (Al_2O_3) and Cu as base nanoparticles through a circular cylinder. The fluid inside the tube is acted by an oscillating pressure gradient and an external constant magnetic field. The whole study is based on a mathematical model that includes Caputo fractional-order derivatives. Solutions for the blood velocitie, blood temperature distribution, and blood concentration distribution are obtained through the Laplace transform and expressed by the Wright function. Effects of the fractional-order parameter, magnetic field, the magnetic parameter M, the Grashof numbers Gr and Gm, the dimensionless time t, and the Prandtl parameter Pr are addressed using numerical simulations. Results show that the applied magnetic field reduces the velocities of the fluid and particles. However, under long time intervals, particles seem to be accelerated, but their velocity is suitably controlled by the fractional-order parameter.

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1 Introduction

Biomagnetic fluid dynamic (BFD) is a new area of research in mechanical fluid [30]. The transportation of drugs, cell separation devices, control of bleeding during surg-

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eries and treatment of cancer tumors are efficiently done by biomagnetic fluids [12,21]. The inclusion of nanomaterials in the base fluid is an attracting technique that have been invented by scientists to improve the thermal efficiency [30]. In 1995, nanofluids introduced by Choi and Eastman [6] have received a considerable attention in current times. Nanofluids are essentially a combination of nano-sized objects contained in a fluid, name base fluid, that increases the thermal characteristics due to the collaboration of these nanomaterials [13].

Blood is a biomagnetic fluid, due to the strong presence of the erythrocytes playing the role of magnetic particles and the plasma as liquid carrier. The magnetization of blood can be augmented by adding artificially created nanoparticles to the flow as usually done in drug targeting delivery. One of the work addressed to the aspect of drug targeting was done in [11], where they studied the mechanism of magnetic targeting of carrier particles in the microvasculature for therapeutic uses. Biofluids should be considered as a medium that exhibits high electrical conductivity [10,15], especially when additional magnetic particles are injected in blood to carry drugs. This fact may justify the use of the principles of magnetohydrodynamics (MHD) [8, 10], which rely on the generation of Lorentz forces, in elaborating suitable mathematical models. Sharma *et al.* [25] studied for example the effect of external uniform magnetic field on flow parameters of both blood and magnetic particles based on a mathematical model using the MHD approach. Magnetic drug targeting (MDT) technique was used by Bose and Banerjee [3], where fluid hydrodynamic and MHD principles were coupled to track the magnetic particles under the effect of a magnetic field. Kefayati [17] studied the effect of a magnetic field on non-Newtonian blood flow between two-square concentric duct annuli, and further extended the study to non-Newtonian blood flow in a cavity driven by the motion of two facing lids [18]. In both cases, the results showed that the increment of Reynolds number augments the magnetic field effect on the flow of blood.

MDT is an interesting topic of research to be continued for discovering a better and improved method of drug delivery by employing a new factor based on the fractional calculus. Fractional calculus is one of the generalizations of classical calculus and it has been successfully applied in the various fields of science and engineering [5, 27–29, 31]. The studies [22, 28, 29] examined the effects of the fractional-order and magnetic fields on the flow of blood in a cylindrical domain by replacing the time derivative of order one with the Caputo fractional derivatives.

Motivated by the above investigations the present paper aims to study the combined heat and mass effects on an unsteady model of MHD blood flow. The objective of the present work is to study a mathematical model of blood flow in presence of magnetic particles and blood temperature, using Caputo fractional derivatives [4, 23]. External magnetic effects are considered and the blood in the vessel is assumed to be acted by a periodic pressure gradient. The mathematical model, which includes Caputo fractional derivatives, is generalized from the model containing integer derivatives, and solutions for the blood velocity, particle velocity and blood temperature are obtained by only making use of Laplace transform. Since Laplace transform expres-