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Mathematical Insights into Substance Addiction and Abuse Dynamics via Global Nonlocal Operators

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Abstract. This paper introduces a novel mathematical model for capturing the complex dynamics of drug abuse within populations. Departing from conventional methodologies, the model employs global derivatives to integrate non-local effects, thereby offering enhanced insight into the spread and evolution of drug abuse. The stability analysis and numerical simulations conducted in this study reveal critical thresholds and dynamic behaviors that are instrumental in understanding the persistence and potential escalation of abuse within communities. Numerical simulations also demonstrate the long-term behavior for different orders of α, and the effects of the function $g(x)$ are presented, further elucidating the intricate interplay of factors that govern the system's dynamics. These findings not only shed light on the underlying mechanisms driving the temporal and spatial patterns of drug abuse but also provide valuable guidance for designing effective intervention strategies aimed at mitigating its spread. By systematically manipulating key parameters, the model serves as a powerful tool for exploring the driving factors behind drug abuse diffusion and control. The insights gained from this research have significant implications for public health policy, offering a rigorous mathematical framework to inform targeted efforts in curbing the epidemic of drug abuse.

AMS subject classifications: 34A08

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1 Introduction

The impact of illegal drug use remains significant, resulting in the loss of numerous valuable lives and productive years. In 2022, drug-related deaths were estimated at around 11.8 million [10]. Globally, between 172 million and 424 million individuals aged 15 to 64 were estimated to have engaged in illicit drug use, [3]. Illicit drug use refers to the non-medical consumption of substances prohibited by international law, such as amphetamines, cannabis, cocaine, heroin, diploids, and MD-MA. The risks

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of premature death and health problems associated with illegal drug use depend on factors such as the amount consumed, frequency of use, and method of administration, [12,17,19].

Studies have shown that factors such as age, gender, socio-economic status, and mental health can all play a role in drug use and related health outcomes [1,2,10]. For example, younger individuals, males, and individuals with lower socio-economic status are more likely to use drugs, and are also at higher risk for adverse health outcomes. Additionally, individuals with mental health conditions, such as depression and anxiety, are more likely to use drugs, and drug use can also exacerbate existing mental health problems [12,17].

Mathematical models have been used to better understand the dynamics of drug use and its impact on population health. These models can help researchers identify key drivers of drug use, as well as the relationships between drug use and various health outcomes, [20]. In recent years, there has also been growing interest in using fractional calculus to model drug use, as this approach allows for the exploration of complex, non-linear relationships, [9, 15, 21]. These operators provides a more advanced mathematical approach that can capture the intricate dynamics and behaviors involved since traditional calculus assumes exponential growth or decay and lacks the ability to capture long-term memory effects and non-exponential behaviors [5,7,15].

1.1 Motivation

Substance addiction and abuse are complex public health challenges influenced by a delicate interplay of biological, psychological, and environmental factors. These dynamics evolve over time and across different populations, making it essential to develop mathematical models that can capture both local and global influences on addiction and recovery processes. Traditional models, often based on classical or fractional derivatives, may not fully account for the long-range dependencies and spatial diffusion patterns observed in real-world substance abuse cases. To address these limitations, a more comprehensive framework is needed. One that integrates non-local effects to better reflect the progression of addiction, the role of external interventions, and the impact of social and environmental influences.

2 Fractional operators and global rate of change

Here, we present some well known fractional operators.

Definition 2.1. *The Riemann-Liouville fractional-order integral operator is* [22]

$$I_t^n f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{n-1} f(\tau) d\tau. \tag{2.1}$$