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Mbayu's Model of Patriotism: Integrating Sports, Academia, and Socio-Cultural Influences

Abdon Atangana[1,2,3,*]

[1] *Institute for Groundwater Studies, Faculty of Natural and Agricultural Sciences, University of the Free State, Bloemfontein 9301, South Africa.*

[2] *Department of Medical Research, China Medical University Hospital, China Medical University, Taichung, 00A71, Taiwan.*

[3] *IT4Innovations National Supercomputing Center, VSB–Technical University of Ostrava, 708 00 Ostrava-Poruba, Czech Republic.*

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Abstract. Patriotism is a changing, complex force inside a nation, whose manifestations are determined by factors as diverse as success in sports, scientific discoveries, cultural pride, economic development, and the presence of national heroes. This study develops a generalized mathematical model for patriotism as the sum of these effects, both emotional and enduring. In particular, the model records surges in patriotism after major sporting victories due to the transience of these surges, for instance, after the victory of the Indomitable Lions or the boxing exploits of Francis Ngannou. In contrast, the model also underlines the long-lasting characteristics of patriotism shaped by scientific progress, such as a Cameroonian being ranked high in mathematics, cultural legacy, infrastructure expansions, and economic development. The sports effect is a series of ephemeral spikes in patriotic sentiment triggered by importance and emotional resonance. In contrast, both the science and infrastructure effects are slowly cumulative processes that build a longer foundation of national pride. These effects, alone and in combination, are shown in the visualizations of their contributions to patriotism over time. This work emphasizes the importance of acknowledging and effectively harnessing these myriad influences for a strong, cohesive nation. The insights are actionable, helping leaders to stimulate a rational yet productive sense of patriotism that is engaged with national development, starting with emotional stimuli but progressing to more permanent investments.

AMS subject classifications: 00A71, 34A05, 34A34

Keywords: Mathematical models, patriotism, emotional patriotism, enduring patriotism, sports achievements, scientific advancements, infrastructure development, cultural heritage, economic growth, national unity.

*Corresponding author. *Email:* AtanganaA@ufs.ac.za (A. Atangana)

1 Introduction

Patriotism is not only the heart of national integrity, but it is also a prerequisite for any society to be united, to have a collective identity, and to develop. For Cameroon a multicultural country that is rich in both natural resources and human capital, patriotism is a both need and a challenge. These pursuits are counterbalanced with socio-political struggles like tribalism, economic disparities and regional tensions, but the country has seen some sense of unity through its sports, achievements in science and cultural preservation in the international sphere. This dynamic shapes the strength and direction of patriotism and speaks to the complexity of factors that shape national pride [2–4, 7]. This study presents a mathematical model which can help quantify and analyze some important drivers of patriotism in Cameroon. Some of these included sporting achievements, global recognition of sport personalities including Francis Ngannou; scientific advance in the promotion of global mathematicians as exemplified by Prof. Abdon Atangana; the influence of cultural festivals, socio-economic development and also on the deleterious effects of tribalism. The model could be able to capture these dynamics, rendering the temporal transformation of patriotism discernible, and highlighting the relative impact of each factor in shaping the overall trajectory. This work aims to provide policymakers with a scientific basis on which to create impactful programs. In seeking to invest in areas of pride that can foster national unity while diminishing the numbing hatred that divides us, and building on the gooey aftermath of the summer to invest in the continuing benefits of sport, science and cultural achievements, such strategies would serve a useful purpose.

2 A Mbayu's mathematical model of patriotism

2.1 Motivation

This passion for national pride must serve as the basis for national unity and socio-economic progress in Cameroon. Longevity, however, has run hand-in-hand with the culture of corruption, there is a rich tapestry of history in Cameroon which has produced cultural diversity, elite performance and intellectual brilliance, but these pillars have failed to take root as a ubiquitous nationalistic tree. Times of pride as a people, like when Francis Ngannou won in boxing or when the world acknowledged the talents of Cameroonian mathematics scientists. These moments should hold promise of uniting Cameroon as a country. Still, tribalism, minimal public investment in science and education and socio-economic inequality remain obstacles to a coherent national identity. This paper is born out of two pressing needs: to discover and measure what accounts for patriotism in Cameroon and to supply policymakers with a framework for evidentially based decision making. National documents are distinguished through the nature of their positive effect, while national divides, such as tribalism, provide insight into review-able ideas to engender patriotism.

2.2 Mathematical model

A sense of pride in belonging in these events and achievements among people is the foundation of national identity and unity which shapes patriotism, a diverse set of events that shape patriotism. These span from sweeping victories in international competitions or enduring collaborative completion of major projects to hosting the world's most prestigious sports events that glories the country and unites the citizens in pride and celebration. Recognition of a nation's cultural and artistic talents of its artists, musicians and writers on the world stage only reaffirms the scope and wealth of its heritage and traditions [1, 5, 6, 8]. Likewise, great technologies and sciences, as well as world-class scientists and other scientific awards, represent the science and technology competitiveness of a country, generating admiration and belief in the future of a country. Any wins on the political or diplomatic front like hosting powerful summits, or solving crises with strong leadership showcase national strength and place the country within the international context. Historical commemorations, independence or monumental achievements celebrated together are memorialization of what the citizens have gone through as a shared history, and large-scale economic celebration, infrastructure works, trade are seeds or symbols that demonstrate the will and the possibility for a better life. During seismic national crises, acts of humanitarianism and social unity further cultivate that spirit of compassion and collective resilience. Military victories and participation in peacekeeping efforts are indication of a country's commitment to sovereignty and international stability, whereas educational achievements and advancements in research demonstrate its investment in future generations. A nation's obligation to protect its natural heritage for posterity is underscored by environmental conservation initiatives and leadership in sustainability. And lastly are the responses to challenges, be they internal or external to the society, which unite the people towards a common goal, be it fighting corruption or poverty; these are living proof of the lasting strength of unity and collaboration. Such features act as great contributors, together enhancing a strong sense of patriotism that contributes an even more profound force, translating into national attachment, one of the strongest drivers of patriotism.

2.3 A mathematical model

Here we start with a simple mathematical model. This model will represent the effect of sport on patriotism. The model will involve identifying some key factors that are known to influence patriotism and how they are affected by sports. Let $p(t)$ be the patriotism level. This is a measure of national pride changing as a function of time. $\delta(t)$ be the sport events influence. This is the impact of sporting events on patriotism, this will include, the frequency type in international competitions. $M(t)$ is the media influence. The role of media coverage in amplifying the connection between sports and patriotism.

$E(t)$ social engagement: Discussions estimated by sports events and community activities.

$A(t)$ national achievement: This is about the success of athletes or teams on a national continental or international level. A simple mathematical model is given by

$$\frac{dp(t)}{dt} = \alpha\delta(t) + \beta M(t) + \gamma E(t) - \lambda p(t). \tag{2.1}$$

Here α, β and γ are factors representing the sensibility of patriotism to sports events, social engagement and media influence respectively. λ is the natural decay of heightened patriotism over time. We shall note that for sports events influence $\delta(t)$,

$$\delta(t) = \sum_{i=1}^{N} \big(w_i A_i(t)\big), \tag{2.2}$$

where $A_i(t)$ is the achievement event i and w_i is its weight, reflecting its importance for example a world cup against local tournament.

Media influence $M(t) = h_1$ (media coverage intensity, sentiment analysis of the coverage).

$E(t) = h_2$ (event participation rate, discussions on social media). Now to highlight how the win of sports figure will impact patriotism in a country, we can simplify the above equation to simple one. Here, we will focus on how such a win can impact national pride and translate into measurable patriotism $p(t)$

$$\frac{dp(t)}{dt} = \alpha_1\delta(t) + \alpha_2 M(t) - \lambda p(t), \tag{2.3}$$

where $\delta(t)$ influence the sports victory for example a world cup win, continental cup win, boxing match win or other tournaments. $\delta(t)$ accounts for a pulse function to represent a significant event like Ngannou's win. The above equation can be solved by using the integrating factor since $\delta(t)$ and $M(t)$ are known functions. We let

$$\mu(t) = \exp(\lambda t), \tag{2.4}$$
$$\exp(\lambda t)\frac{dp(t)}{dt} + \lambda\exp(\lambda t)p(t) = \exp(\lambda t)\big(\alpha_1\delta(t) + \alpha_2 M(t)\big)$$

such that

$$\frac{d}{dt}[\exp(\lambda t)p(t)] = \exp(\lambda t)\big(\alpha_1\delta(t) + \alpha_2 M(t)\big). \tag{2.5}$$

Integrating, we get

$$\exp(\lambda t)p(t) - \exp(\lambda t_0)p(t_0) = \int_{t_0}^{t} \exp(\lambda\tau)\big(\alpha_1\delta(\tau) + \alpha_2 M(\tau)\big)d\tau. \tag{2.6}$$

That is to say,

$$p(t) = p(t_0)\exp\big(\lambda(t-t_0)\big) + \exp(-\lambda t)\int_{t_0}^{t} \exp(\lambda\tau)\big(\alpha_1\delta(\tau) + \alpha_2 M(\tau)\big)d\tau. \tag{2.7}$$

For a special case, we set $\delta(t)$ and $M(t)$. This case can be justified by the following arguments.

Consistent sporting engagement or infrastructure. Here for $\delta(t)$, the country consistent by hosts a similar level of sporting activities or even a country achieves stable performance in international sports event over a certain period, thus $\delta(t)$ can be considered as a constant. A practical example is like Cameroon consistently qualifies for and performs in major tournament like the Africa Cup of Nations or the FIFA World Cup. In this case the patriotism boost from mentioned event stays relatively unchanged year after year. For $M(t)$, one will consider the stable media engagement. This is when the most national influential media constantly broadcasts patriotic content such as national achievements, including major sports victories, in this case $M(t)$ could be considered as a constant. In these cases

$$p(t) = \exp(-\lambda t)\left[\exp(\lambda t_0)p(t_0) + \frac{\alpha_1\delta_0 + \alpha_2 M_0}{\lambda}\big(\exp(\lambda t) - \exp(-\lambda t_0)\big)\right]. \quad (2.8)$$

Here the decay term $-\lambda p(t)$ presents a stabilizing effect since it pulls $p(t)$ back toward a steady-state over time.

$$p_{eq} = \frac{\alpha_1\delta + \alpha_2 M}{\delta}. \quad (2.9)$$

If $\lambda > 0$, the equilibrium is stable

$$\lim_{t\to\infty} p(t) \quad \to \quad p_{eq}. \quad (2.10)$$

If $\delta(t)$ and $M(t)$ are time-dependent for example they can be periodic oscillatory, exponential, then patriotism remains unchanged. This sets as a fundamental level. The solution of the proposed model will be comprehensively illustrated and detailed through the visual representations provided in Figs. 1 and 2, presented below. These figures aim to depict and analyze the significant levels of patriotism that arise as a result of remarkable sports achievements, such as the recent victory of Ngannou, which has served as source of immense pride and unity among people.

The graphs also show a significant drop in the level of patriotism associated with sports victory over time. This is largely because of the type of patriotism born from such occasions, which, though it can stir and galvanize the population and amplify a sense of pride in the nation, is often ephemeral. The temporal nature of the effect is primarily due to the fact that the patriotism induced by sports victories, while strongly felt in the moment,is fundamentally sentimental. It is fueled by the deluge of euphoria, solidified by the singular moment of collective jubilation by a sport coming together around a big win too close to home like Ngannou's stunning takeover. Yet, with the fading of emotional intensity, the unifying effect weakens, and society's comforting social glue becomes increasingly routine. It is only ever so briefly that the world displays a grateful commonality of purpose, a sort of "do not play with us" sentiment and filtering out these moments is the key to finding them again and building something sustainable out of them. These graph therefore demonstrates the

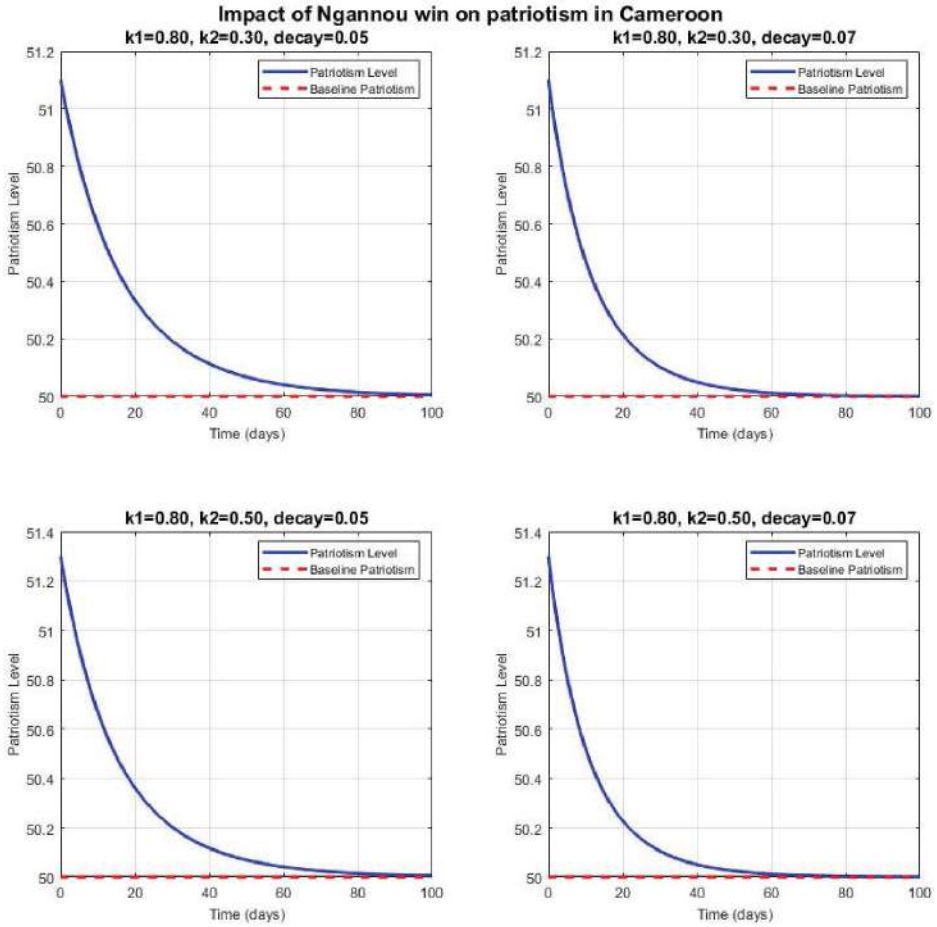

Figure 1: Ngannou's victory sparked a surge of national pride that, while intense and widespread at first, exhibited an exponential decay over time its influence on patriotism gradually diminishing as public attention shifted and everyday concerns resumed.

transient nature of the boost to patriotism at the same time highlighting the influence of sustaining such effects through reinforcement strategies, for example media coverage, public events and community engagements. The proposed patriotism model, although comprehensive and considering various dynamic interactions contributing to the development of patriotism, has several limitations: It simplifies the extremely complex and subjective nature of patriotism into several measurable classes without accounting for psychological, emotional, and historical factors, nor for individual or regional variations; it assumes constant parameters such as α and γ, which could change over time as a result of changes in societal dynamics, political leadership, or economic conditions; it omits the effects of external shocks or crises such as wars, pandemics, or global economic crises that could drastically change one's patriotism; and over-simplifies the negative impacts class, leaving out factors such as political polarization or international pressures; it assumes linear interactions between classes, when these relationships could contain nonlinear feedback or diminishing returns; it

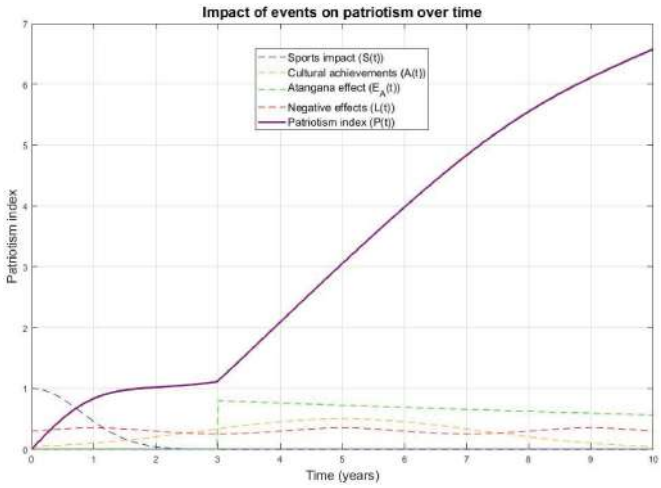

Figure 2: Temporal evolution of patriotism influenced by key national events: Impact of sports victories, scientific milestones, and cultural achievements on collective national sentiment.

is based on a generalized view of patriotism that might not apply universally across various countries or cultures, ignoring cultural and historical differences; it ignores contributions at the individual level and grassroots movements that can influence patriotism; and it the dynamics of patriotism as simplistically, under-representing the complications of feedback loops and non-linearity, where the increase of one type of patriotism could bring impact to another in unexpected ways. The model also has its merits in that it can capture the interplay across different species of patriotism and how one domain (e.g. social pride) can have a positive effect on others, for instance, scientific pride and economic pride. It represents a network of patriotism, realizing that victories in one sector (such as economic growth or scientific advancement) would enrich national pride in other sectors. Furthermore, the model considers the effects of dynamic negative factors, like corruption and injustice, on patriotism, showing the progressive erosion of patriotism across several classes due to its decay from an imbalanced situation. Lastly, it intends to mirror the diverse and complex nature of patriotism, acknowledging how patriotism is affected by national successes as well as failures and individuals that oppose national identity, thereby providing a conceptual basis for examining patriotism as seen through the social ladder within the varied cultural landscape of countries like Cameroon. It also offers an important perspective for understanding the evolution of something as generally positive as patriotism, as in this case, it both helps analyze the turn of one society but also brings more light into how complex societal dynamics can iterate.

3 A model with academic achievements

Here we present a simple model with an academic achievement. For example the impact of a national figure that has achieved global recognition in a field like mathe-

matics. This indeed introduces a significant factor to the model. This is also a factor that elevates patriotism in multiple ways, particularly by inspiring pride in intellectual achievement and showing the international relevance of a nation's talent. We shall now present a simple mathematical model. The academic achievement effect can be modeled as a time-dependent variable. $E_A(t)$ represents the cumulative impact of such achievement on national pride. The impact can be integrated into the overall patriotism index $p(t)$ as follows:

$$\frac{dp(t)}{dt} = \gamma\delta(t) + \lambda A(t) + \xi E_A(t) - \Omega L(t). \tag{3.1}$$

Here $\delta(t)$ is the impact of sports events $A(t)$ is contributions of cultural and artistic achievements. $E_A(t)$ scientific achievements, $L(t)$ is the negative impact for example economic struggles an so on. γ, λ, ξ and Ω are weights representative the relative importance of each factor.

The characteristics of academic achievement effect, for example, being ranked number 1 by the Stanford list of top 2%. It is a global recognition, being ranked number 1 in the world in mathematics indeed fosters national pride by associating the country with intellectual excellence. An inspiration for education since such recognition can indeed becomes a source of inspiration to younger generations to pursue academic excellence, strengthening the national identity around education.

International relations: this positions the country as a global leader in academia, this is also enhancing its soft power and influence. This effect tends to persist as time passes by, in particular when celebrated widely with the country via media, education reforms, and national events. Noting that for the academic achievement like being ranked number 1, the growth function with a term to reflect the long-term but bounding the impact is considered.

$$p_A(t) = A\big(1 - \exp\big(-\beta(t - t_0)\big)\big)H(t - t_0), \tag{3.2}$$

where A represent the maximum contribution to patriotism, this reflect the global impact of being ranked number 1 or receiving a noble price or international award. β is the growth rate of the effect, this factor shows how quickly the recognition spreads and impacts national pride. Indeed, scientific achievement such as a noble price, being ranked number one in a specific subject in the world, foster a long-term and inclusive sense of patriotism by leaving a lasting legacy via advancements in education, research and technology enhancing the global reputation of national institutions inspiring future generations and a sustained cultural and intellectual impact that will transcends the fleeting emotional response typically associated with sports victories which powerful are often tied to short-lives events and cyclical competitions. The model can be enhanced by introducing more factors including cultural pride, diaspora influence, economic stability, political leadership, historical events, educational awareness. In this case the model become

$$\frac{dp(t)}{dt} = \alpha\delta(t) + \beta A(t) + \gamma C(t) - \delta E(t) + \eta D(t) + \xi H(t) - \mu L(t). \tag{3.3}$$

Here $A(t)$ is the academic accomplishments like being ranked number 1 in mathematics. $D(t)$ is the diaspora influence, $H(t)$ is historical celebrations or event. $L(t)$ losses from political unrest or societal challenges. $C(t)$ is cultural contributions, $\alpha, \beta, \gamma, \delta, \eta, \xi$ and μ are sensitivity coefficients. The numerical simulations are presented below (Fig. 2).

The simulation shows that the model integrates short-term spike representing sports wins, with long-term trends representing academic achievements while capturing the social economic and cultural landscape of a country. The simulations illuminate the nuances of patriotism in Cameroon through passion for sports,driven by the nation's victories within the same context and through an ingrained call to seize academic achievements through steadfast hard work as contributions to Cameroon's success across time. It embodies the idea that the collective identity of the Cameroonian people is formed over time through their persistence in rooting for athletes who stand shoulder stand with defenseless humans on the world stage, as well as by praising and rewarding other scholars and innovators whose endless work creates change in the long term. It embodies signing a nation answerable about the rule of sports highlights not simply each day unify moments, yet additionally the longer term educational accomplishments of scholars preparations its several establishments. These forms of patriotism, quite simply, represent the essence of Cameroon,a nation that brings together the spirit of competition, the ability to dream big and the pride of we are well-rounded thinkers.

A potential new mathematical model can be expressed as

$$\begin{aligned} p(t) = p_{baseline}(t) + A(t) + \delta(t) + C(t) + E(t) + D(t) \\ + V(t) + Q(t) + Y(t) + M(t) + L(t) + S(t), \end{aligned} \tag{3.4}$$

where $D(t)$ is the total patriotism at time t. $p_{baseline}(t)$ is the baseline patriotism level a country. $S(t)$ is sports effect, $A(t)$ is science effect, $C(t)$ is cultural effect, $E(t)$ is economic growth effect, $Y(t)$ is youth empowerment effect, $D(t)$ is diasporal effect, $Q(t)$ is security and units effect, $M(t)$ is media and technological effect and $L(t)$ is infrastructure and hand mark achievement. Noting that

$$S(t) = \beta_S \exp\big(-\gamma_S(t - t_S)\big)H(t - t_0), \tag{3.5}$$

where β_S represents the peak impact of a sport event, the parameter γ_S is a control of decay rate, t_S is the even time and $H(t)$ is the Heaviside function.

$$A(t) = A_0 \exp\big(-\gamma_a(t - t_0)\big)H(t - t_0). \tag{3.6}$$

The parameters are defined similarly as $S(t)$.

$$E(t) = E_0 \ln\big(1 + g(t)\big). \tag{3.7}$$

The function $g(t)$ is the growth rate of economy.

$$C(t) = \sum_j \beta_{c_ij} \exp\big(-\gamma^i_{c_ij}(t - t_{c_ij})\big), \tag{3.8}$$

which is the total effect of various cultural events as a function of time.

$$V(t) = \int_0^t \eta_y R_y(\tau) d\tau. \tag{3.9}$$

Here $R_y(t)$ is the national youth focused initiatives over time.

$$D(t) = \alpha_d R_d(t), \tag{3.10}$$

where the function $R_d(t)$ represents or reflects remittance and advocacy efforts.

$$Q(t) = \beta_q \exp(-\gamma_q t). \tag{3.11}$$

The above function can be improved with reconciliation campaigns.

$$V(t) = \alpha_v \ln\left(1 + \overline{p}_v(t)\right), \tag{3.12}$$

where $\overline{p}_v(t)$ is the number of conservation projects.

$$M(t) = \beta_m I_m(t), \tag{3.13}$$

where this function increases with viral campaigns.

$$L(t) = \sum_j \beta_{l_{ij}} \exp\left(-\gamma_{l_{ij}}(t - t_{l_{ij}})\right) \tag{3.14}$$

for major infrastructure or landmark events. Therefore, the overall patriotism is presented in this case by

$$p(t) = p_{baseline}(t) + \sum_{i=1}^{10} \Lambda_i(t), \tag{3.15}$$

where $\Lambda_i(t)$ are component presented above.

The model considers only positive effects. The absence of negative contributions for some factors may not be realistic for some countries since in many countries, certain aspects mentioned above can also have negative effects on patriotism, depending on laws or circumstances. We shall present a modified model that includes tribalism.

$$\begin{aligned}\frac{dp(t)}{dt} = {} & p_{baseline}(t) + \Lambda_S(t) + \Lambda_A(t) + \Lambda C(t) + \Lambda_E(t) \\ & + \Lambda_Y(t) + \Lambda_D(t) + \Lambda_Q(t) + \Lambda_V(t) + \Lambda_M(t) + \Lambda_T(t).\end{aligned} \tag{3.16}$$

$\Lambda_T(t)$ is the rate of tribalism with positive and negative effects. For positive effect, we have tribal diversity celebrated and integrated into national identity. This indeed can foster national unity and pride. On the other hand, we have negative effects: when tribal divisions lead to favoritism, discrimination, or conflict, this indeed erodes patriotism, here

$$\Lambda_T(t) = \beta_T \frac{I_T(t)}{1 + I_T(t)} - \delta_T \frac{D_T(t)}{1 + D_T(t)}. \tag{3.17}$$

Here $I_T(t)$ measures tribal inclusion effort as equitable policies, celebrations of diversity, and cross-cultural collaborations. $D_T(t)$ measures tribal divisions, such as ethnic conflicts, perceptions of inequality, or favoritism in governance. β_T is the coefficient of positive tribalism, and δ_T is the coefficient of negative tribalism.

$$\Lambda_S(t) = \beta_s \exp\big(-\gamma_s(t-t_s)\big)H(t-t_s) - \delta_s \frac{L_s(t)}{1+L_s(t)}. \tag{3.18}$$

The negative term $L_s(t)$ represent the losses or scandals in sports.

$$\Lambda_A(t) = \beta_a \exp\big(-\gamma_a(t-t_a)\big)H(t-t_a) - \delta_a \frac{R_a(t)}{1+R_a(t)}. \tag{3.19}$$

Here negative contribution could come from lack of infrastructure.

$$\Lambda_C(t) = \sum_j \beta_{c_{ij}} \exp\big(-\gamma c_{ij}(t-t_{c_{ij}})\big) - \sum_l \delta_{c_{il}} \frac{\beta_{c_{il}}(t)}{1+\beta_{c_{il}}(t)}. \tag{3.20}$$

Here $\beta_{c_{il}}(t)$ reflects cultural erosion.

$$\Lambda_E(t) = \alpha_e \frac{d}{dt} \ln\big(1+g(t)\big) - \delta_E \frac{U(t)}{1+U(t)}, \tag{3.21}$$

where $U(t)$ models unemployment or economic instability.

$$\Lambda_Q(t) = \gamma_q^+ Q(t) - \gamma_q^- Q(t), \tag{3.22}$$

$$\Lambda_M(t) = \beta_m I_m(t) - \delta_m \frac{N_m(t)}{1+N_m(t)}. \tag{3.23}$$

$I_m(t)$ measures positive campaign and $N_m(t)$ models negative propaganda or misinformation. Final completed model is given as

$$\frac{dP(t)}{dt} = \overline{p}_{baseline} + (\text{positive terms}) - (\text{negative terms}). \tag{3.24}$$

Here positive terms capture growth factors that even include Victoria achievements, unity and negative terms account for detractors. Here

$$P(t) = \int_0^t [\overline{p}_{baseline} + (\text{positive terms}) - (\text{negative terms})]d\tau + P_{initial} \tag{3.25}$$

with $P_{initial}$ is the starting patriotism level. Numerical simulation are presented below, see Figs. 3-7.

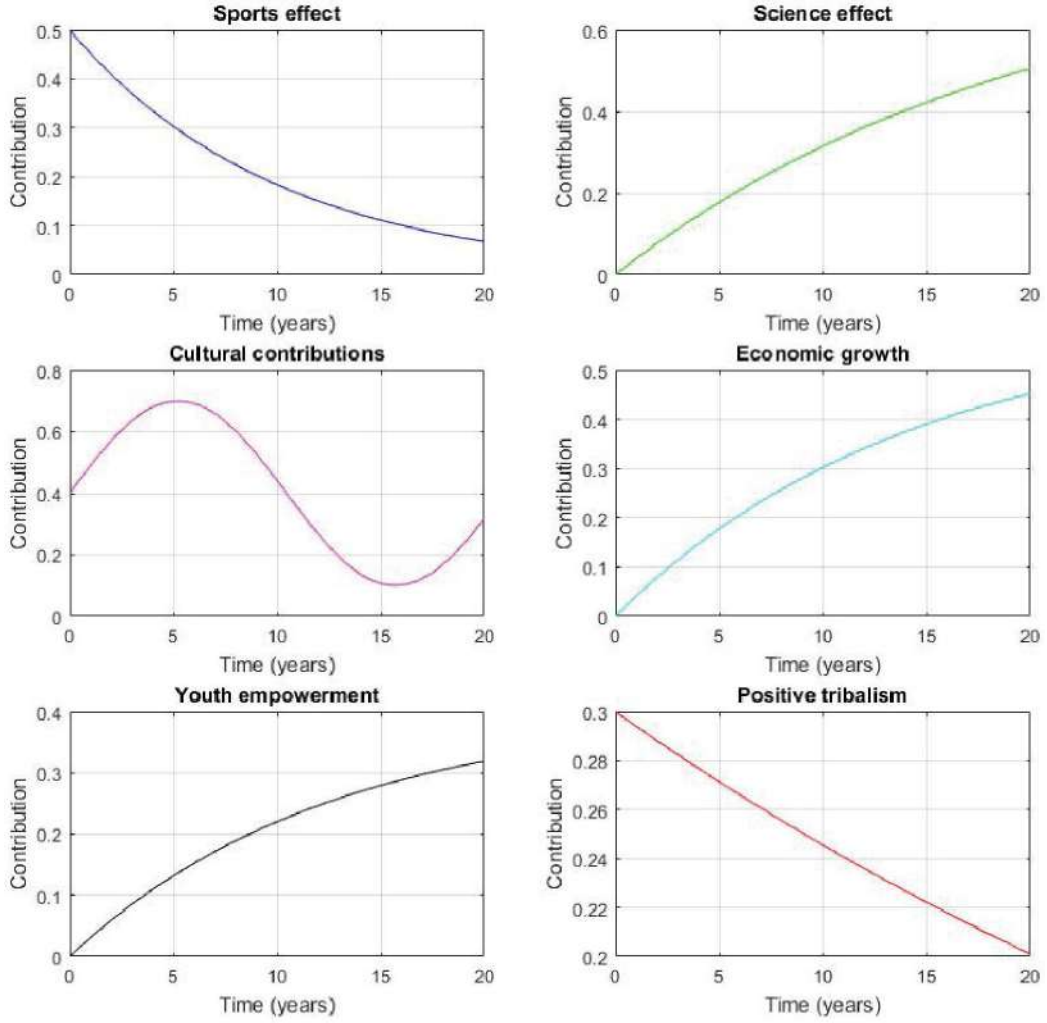

Figure 3: Harnessing the power of sport, science, and culture for economic growth, youth empowerment, and constructive tribal identity.

4 Analysis of a simple model

In this section, we consider the following simplified model:

$$\frac{dP(t)}{dt} = \alpha_a A(t) + \alpha_s S(t) + \alpha_c C(t) - \alpha_T T(t) + \alpha_E E(t) - dP(t), \tag{4.1}$$

$$\frac{dP(t)}{dt} = 0 \quad \Rightarrow \quad P^* = (\alpha_A A^* + \alpha_s S^* + \alpha_c C^* - \alpha_T T^* + \alpha_E E^*)\frac{1}{d}, \tag{4.2}$$

S^*, A^*, T^* and E^* are the steady-state values of the mentioned factors.

1) If $\alpha_A A^* + \alpha_s S^* + \alpha_c C^* + \alpha_E E^* > \alpha_T T^*$, then P^* will be stabilized at a positive value. This means patriotism has reached a steady positive level which indicate a well-managed balance of factors.

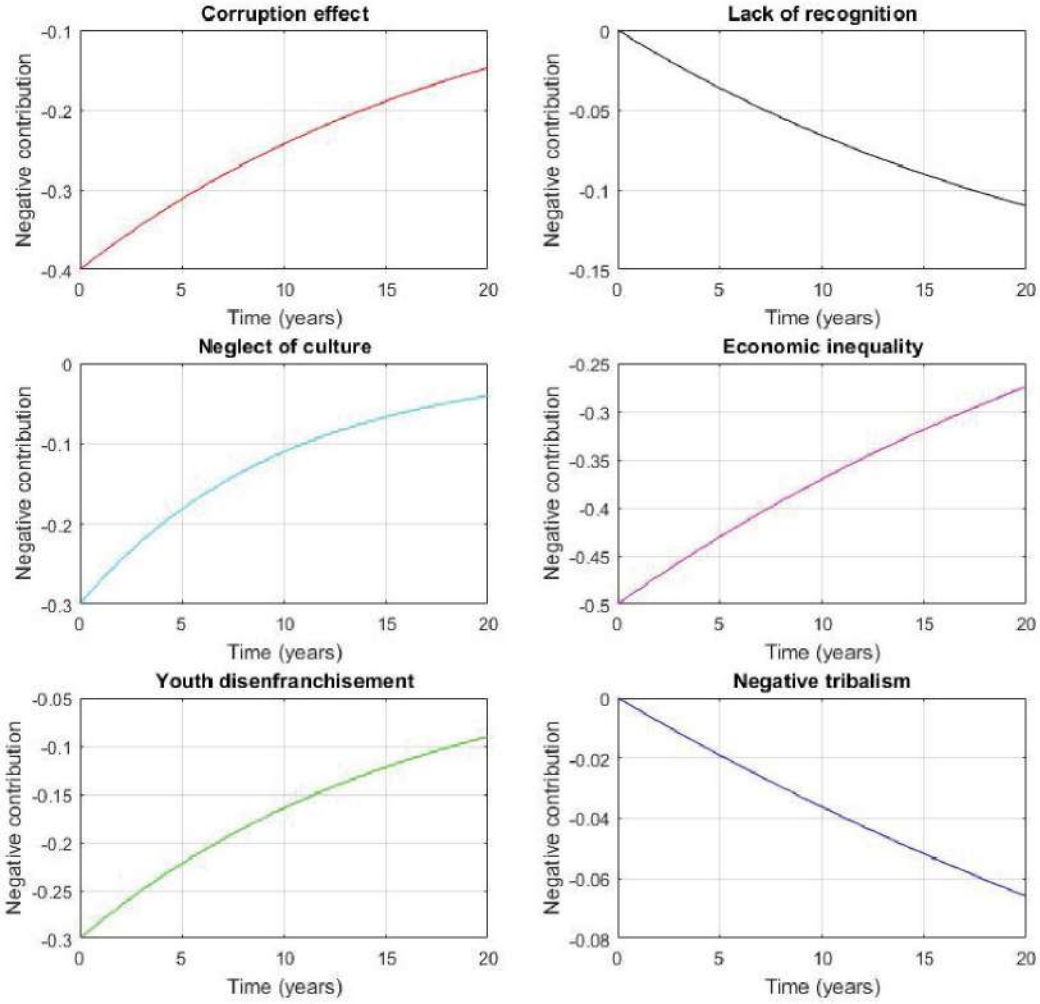

Figure 4: Illustration of key sociopolitical and economic factors hindering national development, including corruption, lack of recognition and cultural neglect, economic inequality, youth disenfranchisement, and negative tribalism all contributing to systemic instability and underdevelopment.

2) If $\alpha_T T^* > \alpha_A A^* + \alpha_s S^* + \alpha_c C^* + \alpha_E E^*$, then P^* will be stabilized at a negative or unacceptably low value. This implies tribalism or other negative effects outweigh positive contributors leading to decline in patriotism. For the time dependent behavior

$$P(t) = P^* + \int_0^t [\alpha_a A(\tau) + \alpha_s S(\tau) + \alpha_c C(\tau) - \alpha_T T(\tau) + \alpha_E E(\tau) - dP(\tau)]d\tau. \quad (4.3)$$

$P(t)$ can growth exponentially (decay or growth). $P(t)$ can oscillate. We note from the steady state that

1) If $\alpha_T > \alpha_A A^* + \alpha_s S^* + \alpha_c C^* + \alpha_E E^*$, then $P^* < 0$, the patriotism collapses.
2) If $\alpha_T \simeq \alpha_A A^* + \alpha_s S^* + \alpha_c C^* + \alpha_E E^*$, then $P^* \simeq 0$, we have neutral patriotism level.

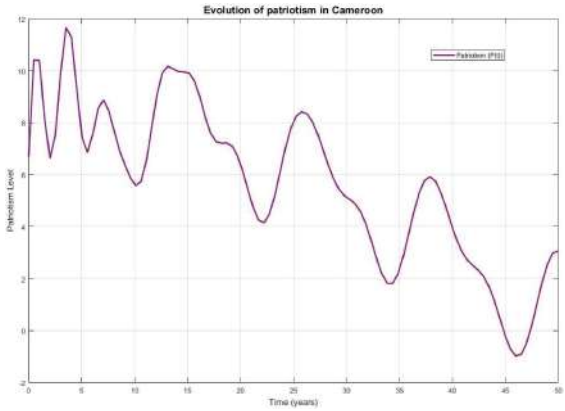

Figure 5: Oscillatory decay pattern illustrating the evolution of patriotism in Cameroon over time, reflecting periods of resurgence and decline influenced by socio-political and historical events.

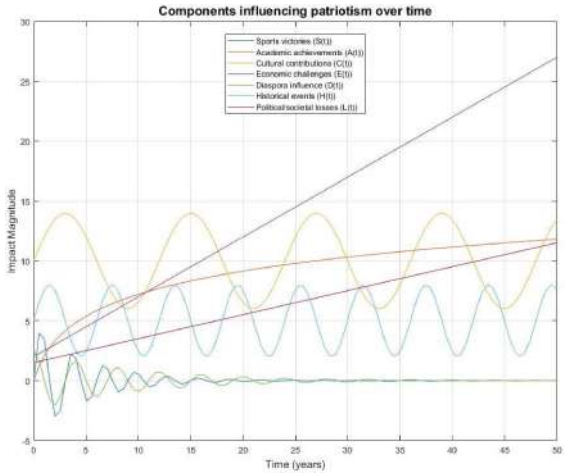

Figure 6: Trends in the key factors influencing patriotism over time, illustrating how social, cultural, economic, and political components contribute to shifts in national identity and allegiance.

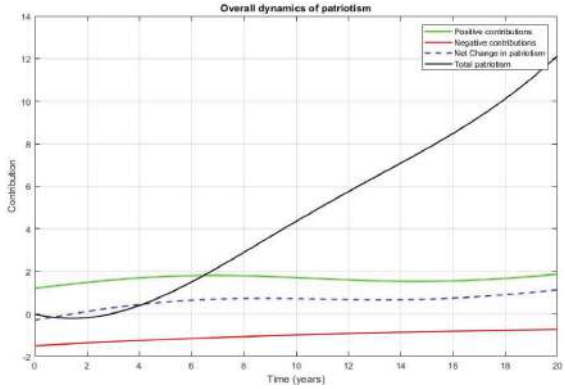

Figure 7: Overall dynamics of patriotism—illustrating how national loyalty evolves through the interplay of cultural identity, historical memory, socio-political influence, and collective experience.

3) If $\alpha_T < \alpha_A A^* + \alpha_s S^* + \alpha_c C^* + \alpha_E E^*$, then $P^* > 0$ stable positive patriotism, then the bifurcation occurs when

$$\alpha_T = \alpha_A A^* + \alpha_s S^* + \alpha_c C^* + \alpha_E E^*. \tag{4.4}$$

a) We will have a saddle-node bifurcation if the contribution from α_s, α_A, α_c, α_E and d belongs against α_T, then the steady state patriotism value can switch between unity and patriotism which is a positive stable state social disunity and tribalism dominance which is a negative or low state.

b) Transcritical bifurcation A_D, α_T causes a critical threshold, patriotism transitions from a positive to a negative steady-state, this indicates dominance for unity to its unity.

Remark 4.1. Increasing α_T for example rising sentiment of tribalism beyond the bifurcation threshold stabilizes patriotism. However, enhancing α_s, α_A, α_c and α_E can stabilize patriotism at higher levels even with moderate tribalism.

4.1 Optimal control analysis

The model will becomes

$$\begin{aligned}\frac{dP(t)}{dt} &= \alpha_a U_A(t)A(t) + \alpha_s U_S(t)S(t) + \alpha_c U_C(t)C(t) \\ &\quad - \alpha_T U_T(t)T(t) + \alpha_E U_E(t)E(t) - dP(t),\end{aligned} \tag{4.5}$$

where $U_A(t)$, $U_S(t)$, $U_C(t)$, $U_T(t)$, $U_E(t) \in [0,1]$ control variables that represent the proportion of investment effort of a government applied to each factor at time t. We suggest and objective function J with objective t_0 maximize patriotism over a fixed time horizon $[0, t_{0b}]$ while minimizing the causes associated to with the controls

$$J = \int_0^{t_{0b}} \left[P(t) - \frac{\lambda_S}{2}U_S^2(t) - \frac{\lambda_A}{2}U_A^2(t) - \frac{\lambda_C}{2}U_C^2(t) - \frac{\lambda_T}{2}U_T^2(t) - \frac{\lambda_E}{2}U_E^2(t) \right] dt, \tag{4.6}$$

where $\lambda_S, \lambda_A, \lambda_C, \lambda_T$ and λ_E are weighting factors that represent the lost of applying each control using the portraying maximum principle, we can solve the optimal control problem as

1) We formulate the Hamiltonian

$$\begin{aligned}H(t) &= P(t) + \lambda_P\{\alpha_a U_A(t)A(t) + \alpha_s U_S(t)S(t) + \alpha_c U_C(t)C(t) \\ &\quad - \alpha_T U_T(t)T(t) + \alpha_E U_E(t)E(t) - dP(t)\} \\ &\quad - \frac{\lambda_S}{2}U_S^2(t) - \frac{\lambda_A}{2}U_A^2(t) - \frac{\lambda_C}{2}U_C^2(t) \\ &\quad - \frac{\lambda_T}{2}U_T^2(t) - \frac{\lambda_E}{2}U_E^2(t),\end{aligned} \tag{4.7}$$

λ_P is the ad-joint variable associated with $P(t)$.

2) Solving the ad-joint equation

$$\frac{d\lambda_P}{dt} = -\frac{\partial H}{\partial P}. \tag{4.8}$$

3) Maximizing the Hamiltonian with respect to control

$$U_S^*(t) = \min\left(\max\left(\frac{\lambda_P \overline{\alpha}_S S(t)}{\lambda_S}, 0\right), 1\right), \tag{4.9}$$

and doing similarity for $U_A^*(t), U_C^*(t), U_T^*(t)$ and $U_E^*(t)$. Here

$$U_T^*(t) = \min\left(\max\left(\frac{\lambda_P \overline{\alpha}_T T(t)}{\lambda_T}, 0\right), 1\right). \tag{4.10}$$

For condition 2 we have that

$$\frac{\partial H}{\partial P} = 1 - \lambda_P d. \tag{4.11}$$

Therefore,

$$\frac{\partial \lambda_P}{\partial t} = \lambda_P d - 1, \tag{4.12}$$

which a first order linear equation with solution

$$\lambda_P(t) = C\exp(dt) + \frac{1}{d}. \tag{4.13}$$

To solve the state equation, ad-joint equation and optimal control simultaneously, we set

$$\begin{aligned} &\frac{dX(t)}{dt} = f\big(X(t), U(t), t\big), \\ &X(t) = \big(C(t), E(t), T(t), S(t), A(t)\big), \\ &U(t) = \big(U_C(t), U_E(t), U_T(t), U_S(t), U_A(t)\big). \end{aligned} \tag{4.14}$$

From the Pontryagin maximum principle

$$\frac{d\lambda}{dt} = -\frac{\partial H}{\partial X}, \tag{4.15}$$

where λ is the vector of ad-joint variable. The control is determined by maximizing or minimizing the Hamiltonian

$$\overline{U}^*(t) = \arg\max_U H(X, \lambda, U, t) \tag{4.16}$$

or

$$\underline{U}^*(t) = \arg\min_U H(X, \lambda, U, t). \tag{4.17}$$

The solution will be obtained via a numerical scheme $[t_0, t_f]$ is divided into M steps size where $X(t_0)$ is the initial condition for stable variables, $\lambda(t_f)$ set terminal conditions for the ad-joint variables $\lambda(t_f)$,

$$\begin{cases} X_{j+1} = X_j + \Delta f(X_j, U_j, t_j), \\ \lambda_{\sigma-1} = \lambda_j - \Delta f \dfrac{\partial H}{\partial X}, \\ U_j^* = \arg\max\limits_{U} H(X_j, \lambda_j, U_j, t_j). \end{cases} \tag{4.18}$$

In case

$$\frac{dX_P}{dt} = \lambda_P d - 1. \tag{4.19}$$

4.2 Numerical simulations and Interpretations

In this subsection, numerical simulations are conducted to examine the behavior of the proposed simple model under different scenarios. The graphical results illustrate how key factors such as sports achievements, academic advancements, economic growth, and societal influences shape patriotism over time. Special attention is given to the transient and long-term effects, highlighting the interplay between short-lived emotional surges and sustained contributions to national pride. These visual representations provide valuable insights into the model's predictive capabilities and its alignment with real-world patriotic dynamics, see Figs.8-12.

5 System of nonlinear equations

Indeed, by introducing a system of nonlinear equations with different classes adds complexity and realism to the proposed model, because it allows to capture different aspects or a group of patriotism contributors, for example social, scientific, cultural or economic. Here each class can be used to represent a subset of the contributing factors and the interactions between these classes can also be modeled explicitly. The classes are suggested below.

$P_S(t)$: Patriotism driven by social factor such that sport and cultural events.

$P_A(t)$: Patriotism driven by achievement in science and education.

$P_E(t)$: Patriotism driven by economic growth, employment and governance.

$P_N(t)$: Negative impact driven by detractors of patriotism like corruption, tribalism and injustice.

The mathematical model is presented below

$$\frac{dP_S(t)}{dt} = r_S \frac{U_S S}{1 + \alpha_S P_S} + \gamma_{SA} P_A - \delta_{SN} P_N - d_S P_S, \tag{5.1}$$

$$\frac{dP_A(t)}{dt} = r_A \frac{U_A A}{1+\alpha_A P_A} + \gamma_{AS} P_S + \gamma_{AE} P_E - \delta_{SN} P_N - d_A P_A,$$
$$\frac{dP_E(t)}{dt} = r_E \frac{U_E E}{1+\alpha_E P_E} + \gamma_{ES} P_S + \gamma_{EA} P_A - \delta_{EN} P_N - d_E P_E,$$
$$\frac{dP_N(t)}{dt} = \beta_T T + \beta_C C_N + \beta_j J + \beta_I I - \eta_N P_S P_A P_E - d_N P_N.$$

The model is made up with growth damping term.

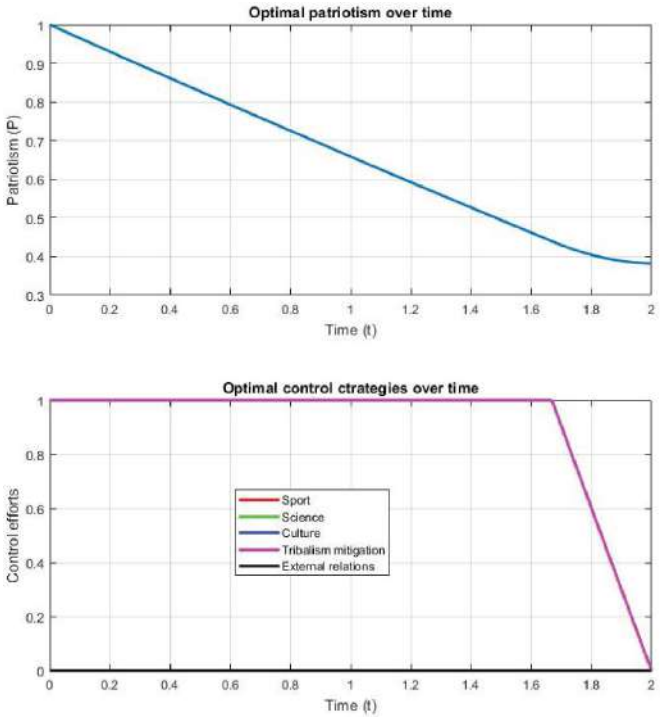

Figure 8: Optimal time-dependent profiles of patriotism and control strategies illustrating the dynamic balance between national loyalty and intervention measures for effective system regulation.

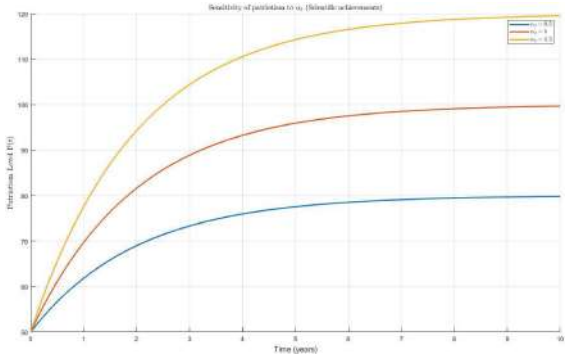

Figure 9: Sensitivity of patriotism to scientific achievements: A quantitative lens on how national pride responds to advances in science and innovation.

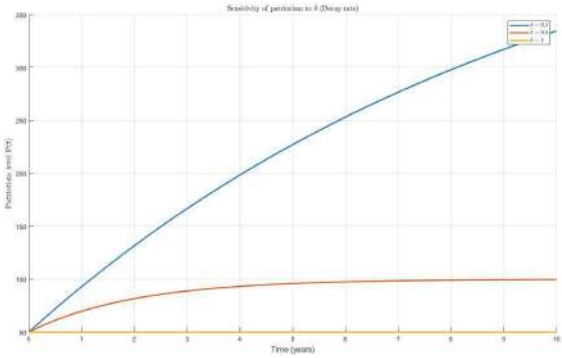

Figure 10: Sensitivity of patriotism to decay rate: Higher decay accelerates loss of patriotic sentiment over time, highlighting the fragility of national attachment under adverse conditions.

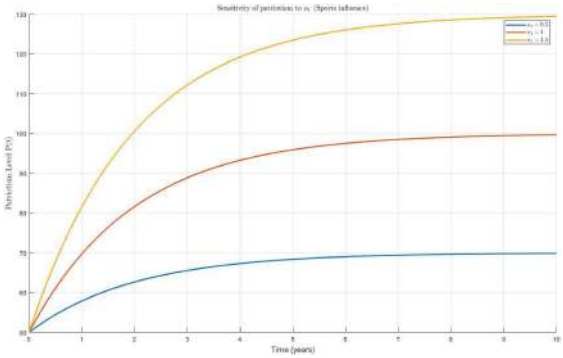

Figure 11: Sensitivity of patriotism to sports performance, highlighting how national pride responds sharply to international sports success or failure.

For growth terms $U_S S/(1+\alpha_S P_S), U_A A/(1+\alpha_A P_A), U_E E/(1+\alpha_E P_E)$ are naturalization effects within each class γ_i, P_i are crossover positive influence for damping terms δ_i, P_N are negative factors reducing patriotism in other classes, $d_i P_i$ are natural decay of patriotism in each class. Negative impacts are (T, C_N, J, I) which are growth from negative factors. $\eta_N P_S P_A P_E$ are dampened by the collective strength of positive classes. T is tribalism, C_N is corruption, J is injustice, I is inequality. r_S, r_A, r_E are growth factor. U_S, U_A, U_E are effort scaling factors that indicate how efficient efforts (S, A, E) translate into patriotism. $\alpha_S, \alpha_A, \alpha_E$ are represent the reducing patriotism levels (P_S, P_A, P_E). γ_{ij} are positive influence, δ_{ij} are negative impacts of P_N on each class. d_i are natural decay of tribalism, corruption, injustice and inequality to P_N.

β_i contribution of T, C_N, I and J to P_N, η_N mitigating effect of positive classes (P_S, P_A, P_E) on negative factors.

Due to non linearity of the model, equilibrium points can be explicit obtained using the Newton's method. However, we have

$$S^* = \frac{(d_s P_S^* - \gamma_{SA} P_A^* + \delta_{SN} P_N^*)(1+\alpha_S P_S^*)}{r_S U_S}, \tag{5.2}$$
$$A^* = \frac{(d_A P_A^* - \gamma_{AS} P_S^* + \delta_{AE} P_E^* + \delta_{AN} P_N^*)(1+\alpha_A P_A^*)}{r_A U_A},$$

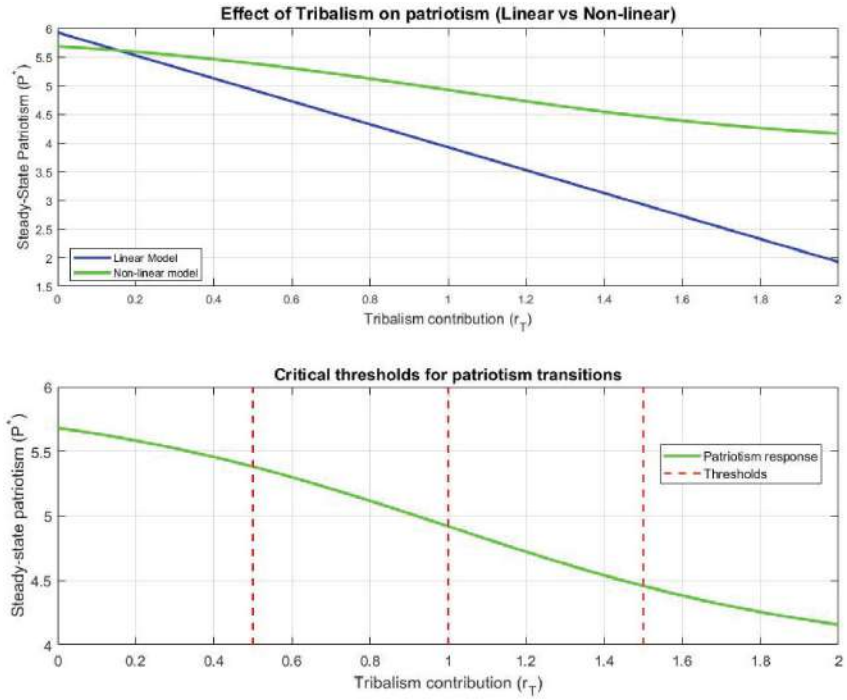

Figure 12: Steady-state analysis illustrating critical thresholds for the transmission of patriotism, highlighting the conditions under which the system transitions from low to high prevalence of patriotic sentiment in the population.

$$E^* = \frac{(d_E P_E^* - \gamma_{ES} P_S^* - \gamma_{EA} P_A^* + \delta_{EN} P_N^*)(1 + \alpha_E P_E^*)}{r_E U_E},$$
$$P_N^* = \frac{\beta_T T + \beta_C C_N + \beta_J J + \beta_I I - \eta_N P_S^* P_A^* P_E^*}{d_N}.$$

We set

$$f_1(P_S, P_A, P_E, P_N) = r_S \frac{U_S S}{1 + \alpha_S P_S} + \gamma_{SA} P_A - \delta_{SN} P_N - d_S P_S, \tag{5.3}$$
$$f_2(P_S, P_A, P_E, P_N) = r_A \frac{U_A A}{1 + \alpha_A P_A} + \gamma_{AS} P_S + \gamma_{AE} P_E - \delta_{SN} P_N - d_A P_A,$$
$$f_3(P_S, P_A, P_E, P_N) = r_E \frac{U_E E}{1 + \alpha_E P_E} + \gamma_{ES} P_S + \gamma_{EA} P_A - \delta_{EN} P_N - d_E P_E,$$
$$f_4(P_S, P_A, P_E, P_N) = \beta_T T + \beta_C C_N + \beta_j J + \beta_I I - \eta_N P_S P_A P_E - d_N P_N.$$

$$J = \begin{bmatrix} \frac{-r_S U_S S \alpha_S}{(1 + \alpha_S P_S^*)^2} - ds & \gamma_{SA} & 0 & -\delta_{SN} \\ \gamma_{AS} & \frac{-r_A U_A A \alpha_A}{(1 + \alpha_A P_A^*)^2} - dA & \gamma_{AE} & -\delta_{AN} \\ \gamma_{ES} & \gamma_{EA} & -r_E U_E E \alpha_E & -\delta_{EN} \\ -\eta_N P_A^* P_E^* & -\eta_N P_S^* P_E^* & -\eta_N P_S^* P_A^* & -d_N \end{bmatrix}. \tag{5.4}$$

5.1 Existence and uniqueness

To apply the fixed-theorem for existence, we reformulate the differential equations into their integral form. We then reformulate the system as

$$\begin{aligned}
P_S(t) &= P_S(0) + \int_0^t \left(r_S \frac{U_S S}{1+\alpha_S P_S} + \gamma_{SA} P_A - \delta_{SN} P_N - d_S P_S \right) d\tau, \\
P_A(t) &= P_A(0) + \int_0^t \left(r_A \frac{U_A A}{1+\alpha_A P_A} + \gamma_{AS} P_S + \gamma_{AE} P_E - \delta_{SN} P_N - d_A P_A \right) d\tau, \\
P_E(t) &= P_E(0) + \int_0^t \left(r_E \frac{U_E E}{1+\alpha_E P_E} + \gamma_{ES} P_S + \gamma_{EA} P_A - \delta_{EN} P_N - d_E P_E \right) d\tau, \\
P_N(t) &= P_N(0) + \int_0^t (\beta_T T + \beta_C C_N + \beta_j J + \beta_I I - \eta_N P_S P_A P_E - d_N P_N) d\tau.
\end{aligned} \tag{5.5}$$

We put

$$T(P) = P(0) + \int_0^t F\big(\tau, P(\tau)\big) d\tau, \tag{5.6}$$

where

$$P(0) = \big(P_S(0), P_A(0), P_E(0), P_N(0)\big), \tag{5.7}$$

$$F\big(t, P(t)\big) = (f_1, f_2, f_3, f_4) \tag{5.8}$$

with

$$P(t) = (P_S, P_A, P_E, P_N). \tag{5.9}$$

f_1, f_2, f_3 and f_4 are continuous since $1+\alpha_S P_S, 1+\alpha_A P_A, 1+\alpha_E P_E$ are positive values.

$$\begin{aligned}
&\frac{\partial f_1}{\partial P_s} = -\frac{r_S U_S S}{(1+\alpha_S P_S)^2} - ds, \quad \frac{\partial f_1}{\partial P_A} = \gamma_{SA}, \quad \frac{\partial f_1}{\partial P_E} = 0, \quad \frac{\partial f_1}{\partial P_N} = -\delta_{SN}, \\
&\frac{\partial f_2}{\partial P_s} = \gamma_{SA}, \quad \frac{\partial f_2}{\partial P_A} = -\frac{r_A U_A A \alpha_A}{(1+\alpha_A P_A)^2} - dA, \quad \frac{\partial f_2}{\partial P_E} = \gamma_{AE}, \quad \frac{\partial f_2}{\partial P_N} = -\delta_{AN}, \\
&\frac{\partial f_3}{\partial P_s} = \gamma_{ES}, \quad \frac{\partial f_3}{\partial P_A} = \gamma_{AE}, \quad \frac{\partial f_3}{\partial P_E} = -\frac{r_E U_E E \alpha_E}{(1+\alpha_E P_E)^2} - dE, \quad \frac{\partial f_3}{\partial P_N} = -\delta_{EN}, \\
&\frac{\partial f_4}{\partial P_s} = -\eta_N P_A P_E, \quad \frac{\partial f_4}{\partial P_A} = -\eta_N P_S P_E, \quad \frac{\partial f_4}{\partial P_E} = -\eta_N P_S P_A, \quad \frac{\partial f_4}{\partial P_s} = -d_N.
\end{aligned} \tag{5.10}$$

This shows that T is Lipschitz conditions exists K such that $P_1 \neq P_2$ solutions of the system

$$|T(P_1) - T(P_2)| < K|P_1 - P_2|, \tag{5.11}$$

for the Lyapunov's direct method, we choose the following function:

$$V(P) = P_S^2 + P_A^2 + P_E^2 + P_N^2. \tag{5.12}$$

Then

$$\frac{dV}{dt} = 2\dot{P}_s P_s + 2\dot{P}_A P_A + 2\dot{P}_E P_E + 2\dot{P}_N P_N,$$

$$
\begin{aligned}
&= 2\Bigg(P_s \left(r_S \frac{U_S S}{1+\alpha_S P_S} + \gamma_{SA} P_A - \delta_{SN} P_N - d_S P_S \right) \\
&\quad + P_A \left(r_A \frac{U_A A}{1+\alpha_A P_A} + \gamma_{AS} P_S + \gamma_{AE} P_E - \delta_{SN} P_N - d_A P_A \right) \\
&\quad + P_E \left(r_E \frac{U_E E}{1+\alpha_E P_E} + \gamma_{ES} P_S + \gamma_{EA} P_A - \delta_{EN} P_N - d_E P_E \right) \\
&\quad + P_N \left(\beta_T T + \beta_C C_N + \beta_j J + \beta_I I - \eta_N P_S P_A P_E - d_N P_N \right) \Bigg) \\
&= 2\Bigg(r_S \frac{U_S S P_s}{1+\alpha_S P_S} + \gamma_{SA} P_A P_s - \delta_{SN} P_N P_s - d_S P_S^2 \\
&\quad + r_A \frac{P_A U_A A}{1+\alpha_A P_A} + \gamma_{AS} P_A P_S + \gamma_{AE} P_A P_E - \delta_{SN} P_A P_N - d_A P_A^2 \\
&\quad + r_E \frac{P_E U_E E}{1+\alpha_E P_E} + \gamma_{ES} P_E P_S + \gamma_{EA} P_E P_A - \delta_{EN} P_E P_N - d_E P_E^2 \\
&\quad + \beta_T T P_N + \beta_C C_N P_N + \beta_j J P_N + \beta_I I P_N - \eta_N P_S P_A P_N P_E - d_N P_N^2 \Bigg) \\
&= 2\Bigg(r_S \frac{U_S S P_s}{1+\alpha_S P_S} + \gamma_{SA} P_A P_s + r_A \frac{P_A U_A A}{1+\alpha_A P_A} + \gamma_{AS} P_A P_S + \gamma_{AE} P_A P_E + r_E \frac{P_E U_E E}{1+\alpha_E P_E} \\
&\quad + \gamma_{ES} P_E P_S + \gamma_{EA} P_E P_A + \beta_T T P_N + \beta_C C_N P_N + \beta_j J P_N + \beta_I I P_N \Bigg) \\
&= 2\big(\delta_{SN} P_N P_s + d_S P_S^2 + \delta_{SN} P_A P_N + d_A P_A^2 + \delta_{EN} P_E P_N \\
&\quad + d_E P_E^2 + \eta_N P_S P_A P_N P_E + d_N P_N^2 \big) \\
&= 2\Lambda_1(t) - 2\Lambda_2(t). \qquad (5.13)
\end{aligned}
$$

So, we get

$$
\frac{dV(t)}{dt} < 0 \;\; \Rightarrow \;\; \Lambda_1(t) < \Lambda_2(t), \quad \forall\, t \in [0, T]. \qquad (5.14)
$$

Alternatively, we can consider the following function:

$$
\begin{aligned}
V(P) = {} & W_s P_S^2 + W_A P_A^2 + W_E P_E^2 + a_1 P_S P_A \\
& + a_2 P_A P_E + a_3 P_E P_N + a_4 P_S P_N, \qquad (5.15)
\end{aligned}
$$

where a_1, a_2, a_3, a_4 coefficients representing the interactions between the chosen populations P_S, P_A, P_E, P_N while W_S, W_A, W_E and W_N are positive weight. We note that

$$
\frac{dV}{dt} = \frac{\partial V}{\partial P_S}\frac{dP_S}{dt} + \frac{\partial V}{\partial P_A}\frac{dP_A}{dt} + \frac{\partial V}{\partial P_E}\frac{dP_E}{dt} + \frac{\partial V}{\partial P_N}\frac{dP_N}{dt}, \qquad (5.16)
$$

$$
\frac{\partial V}{\partial P_S} = 2W_s P_S + a_4 P_A + a_4 P_N, \qquad (5.17)
$$

$$\frac{\partial V}{\partial P_A} = 2W_A P_A + a_1 P_S + a_2 P_E,$$
$$\frac{\partial V}{\partial P_E} = 2W_E P_E + a_2 P_A + a_3 P_N,$$
$$\frac{\partial V}{\partial P_N} = 2W_N P_N + a_3 P_E + a_4 P_S,$$

replacing yield

$$\begin{aligned}\frac{dV}{dt} &= (2W_s P_S + a_4 P_A + a_4 P_N)\frac{dP_S}{dt} \\ &\quad + (2W_A P_A + a_1 P_S + a_2 P_E)\frac{dP_A}{dt} \\ &\quad + (2W_E P_E + a_2 P_A + a_3 P_N)\frac{dP_E}{dt} \\ &\quad + (2W_N P_N + a_3 P_E + a_4 P_S)\frac{dP_N}{dt}, \end{aligned} \tag{5.18}$$

then

$$\begin{aligned}\frac{dV}{dt} &= (2W_s P_S + a_4 P_A + a_4 P_N)\left(r_S \frac{U_S S}{1+\alpha_S P_S} + \gamma_{SA} P_A - \delta_{SN} P_N - d_S P_S\right) \\ &\quad + (2W_A P_A + a_1 P_S + a_2 P_E)\left(r_A \frac{U_A A}{1+\alpha_A P_A} + \gamma_{AS} P_S + \gamma_{AE} P_E - \delta_{SN} P_N - d_A P_A\right) \\ &\quad + (2W_E P_E + a_2 P_A + a_3 P_N)\left(r_E \frac{U_E E}{1+\alpha_E P_E} + \gamma_{ES} P_S + \gamma_{EA} P_A - \delta_{EN} P_N - d_E P_E\right) \\ &\quad + (2W_N P_N + a_3 P_E + a_4 P_S)(\beta_T T + \beta_C C_N + \beta_j J + \beta_I I - \eta_N P_S P_A P_E - d_N P_N) \\ &= \Lambda(t) - \overline{\Lambda}(t), \end{aligned} \tag{5.19}$$
$$\frac{dV}{dt} < 0, \quad \text{if} \quad \Lambda(t) < \overline{\Lambda}(t).$$

5.2 Numerical solutions

In this section, we present the numerical solutions to the system of nonlinear equations that describe the dynamics of patriotism influenced by various contributing factors. The model includes four distinct classes, each representing different aspects that contribute to the overall sense of patriotism within a society. To analyze the model, we employ numerical methods to approximate the solutions to these equations. The results provide insights into how patriotism evolves under different conditions, highlighting the relative importance of social, scientific, economic, and negative factors. The graphical representations generated from these numerical solutions will offer a visual understanding of the model's dynamics over time, demonstrating how each factor influences the collective sense of patriotism. In the following, we will present the numerical solutions below via Figs. 13-19.

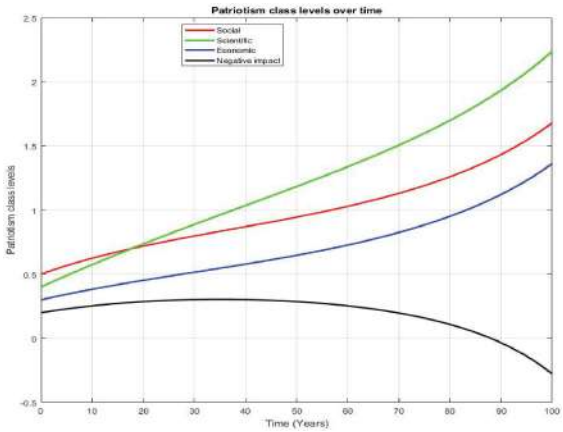

Figure 13: Temporal evolution of patriotism class levels, illustrating the dynamics of critical thresholds required for the transmission and persistence of patriotic sentiment within a population.

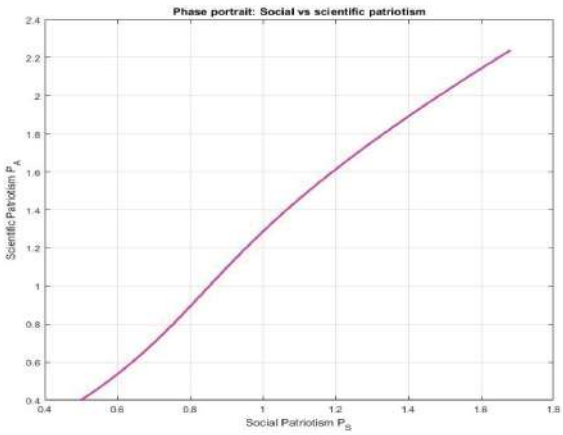

Figure 14: Phase portrait illustrating the critical thresholds for the transmission dynamics between social patriotism and scientific patriotism.

6 Conclusion and recommendations

This research highlights the reciprocal relationship between temporary and permanent attributes that influence national pride. Whereas emotional patriotism based on sports successes can be creating intense and sudden discharges of pride in the country, they are ephemeral and fade over time. In contrast, lasting patriotism creates a sense of national unity, producing a culture based on science, infrastructure development, cultural heritage, and economic growth and extending over generations. Strategically understanding the two dimensions of both can help unify a fragmented society put together. Emotional patriotism can then be reinforced by leveraging successes in sports like the Indomitable Lions and global successes like Francis Ngannou. Sports, especially during such moments of national pride, need to be celebrated publicly and through the media, and the certain governance of sports development and programs for youth, the activities that can pull the young people together,should

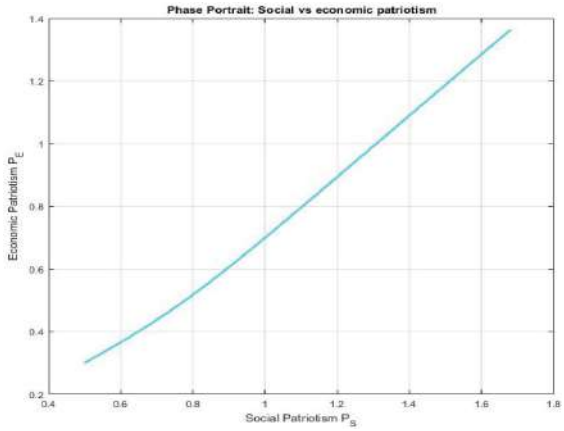

Figure 15: Phase portrait illustrating critical thresholds for the transmission dynamics of social versus economic patriotism.

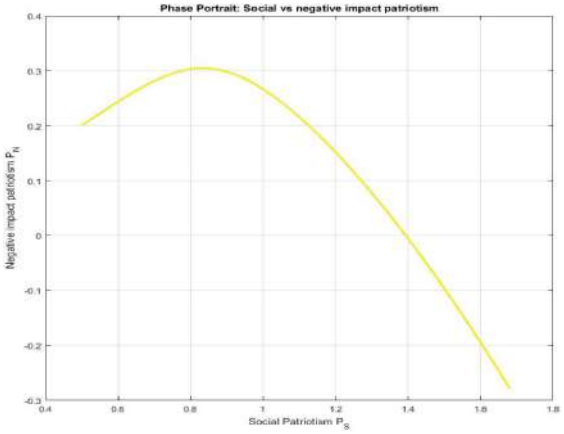

Figure 16: Phase portrait illustrating the relationship between social impact and negative impact of patriotism, highlighting critical thresholds for patriotism transmission.

also be pushed more. Organizing international tournaments, investing in talent development and being able to unite the citizens with high emotions by activating the patriots around a national project can create a kind of ripple effect of patriotism, as these sentiments have far-reaching implications beyond the limitations of the sports basement. In order to build lasting nationalism, state funds should be used to invest in scientific research, infrastructure, cultural preservation and also economic development. Arriving at this realization means that major public celebration of such scientific contributions by Cameroonians is needed to inspire the younger Cameroonians, showcase Cameroon context as a center of innovation and intellectual leadership. Infrastructure investments can lead to improved transportation, healthcare, and education, which directly enhances citizens' quality of life and instilling a sense of national pride in progress! In addition, enhancing and documenting Cameroon's diverse cultural heritage by means of festivals, museums, cultural exchange programs can help further entrench the population's bond with their identity. Patriotism is fleeting, one

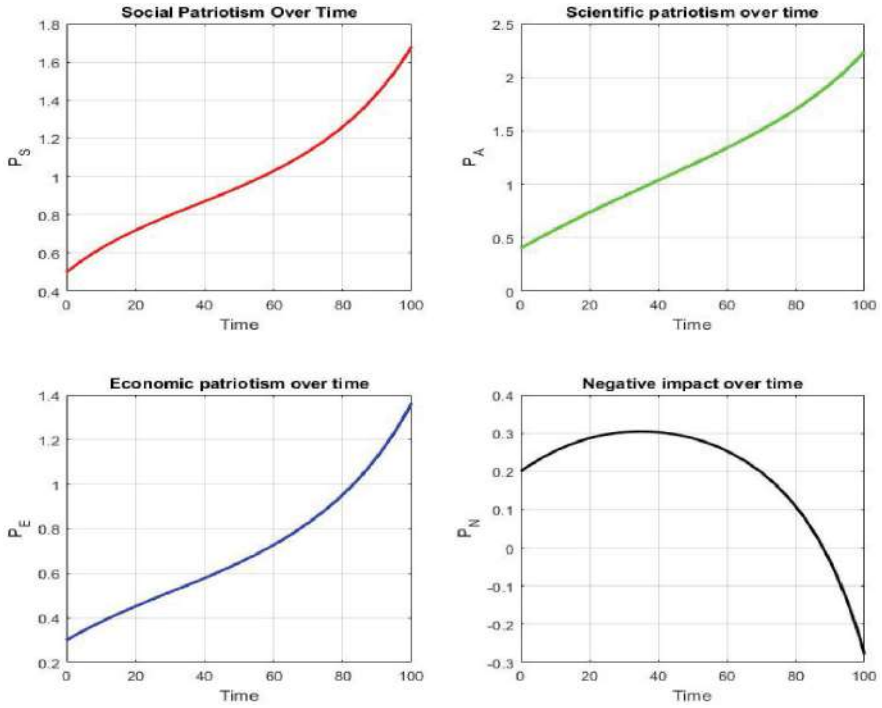

Figure 17: Positive and negative contributions to patriotism over time, illustrating critical thresholds in the transmission dynamics of patriotic sentiment.

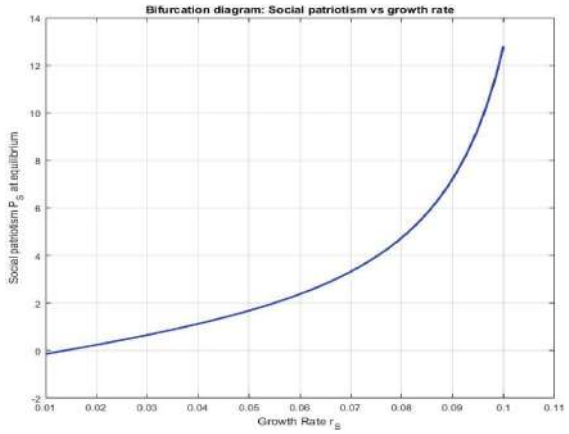

Figure 18: Bifurcation diagram showing the relationship between social patriotism and growth rate, highlighting critical thresholds for the transmission of patriotism.

of those emotions that relies on inspirational figures to bridge the gap to something longer lasting. The government must also use its platform to promote such individuals and promote opportunities of their achievements at national level programs and in educational curriculum and initiate the display of leadership and excellence in all sectors. This can inspire fellow citizens to follow in the footsteps of these role models and help build the nation. Striking a nuanced balances all-important, so as to make

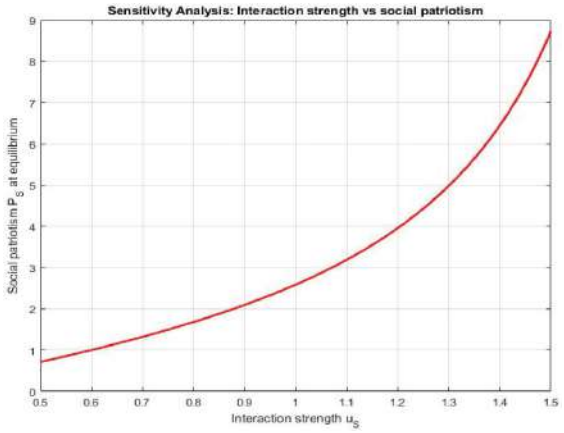

Figure 19: Sensitivity analysis showing the critical thresholds for patriotism transmission as a function of interaction strength versus social patriotism.

the most of the positive features of both types of patriotism. The government needs to align short-term emotional triggers (like sporting events) with long-term developmental projects. Public campaigns celebrating sporting success could also promote scientific achievement or infrastructure success, presenting a full picture of national excellence. Lastly, it is important to keep monitoring and evaluating the impact of these efforts. The government is well advised to develop tools to gauge the level of patriotic sentiment-nationwide surveys and backs surveys of data analytic are all possibilities. Such insight scan inform, adjust policies and programs so they respond to citizens and meet the intended purpose of said policies and programs.

In conclusion, Cameroon's government must understand that emotional patriotism inspired by sport may be fleeting, yet it is a forceful agent of national unity. Sustaining and deepening this patriotism will require long-range investments in science, infrastructure, culture and economic development. Cameroon can develop a deep, sustainable feeling of national pride with the right mix of these approaches that benefits the citizens and encourages progress, resiliency and unity among its people.

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