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A Certain Class of Equi-Statistical Convergence Based on (p, q)-integers via Deferred Nörlund Mean and Related Approximation Theorems

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Abstract. The concept of equi-statistical convergence is more general than that of the well-established statistical uniform convergence. In this paper, we have introduced the idea of equi-statistical convergence, statistical point-wise convergence and statistical uniform convergence under the difference operator including (p, q)-integers via deferred Nörlund statistical convergence so as to build up a few inclusion relations between them. We have likewise presented the notion of the deferred weighted (Nörlund type) equi-statistical convergence (presumably new) in view of difference sequence of order r based on (p, q)-integers to demonstrate a Korovkin type approximation theorem and proved that our theorem is a generalization (non-trivial) of some well-established Korovkin type approximation theorems which were demonstrated by earlier authors. Eventually, we set up various fascinating examples in connection with our definitions and results.

Key Words: Statistical convergence, (p, q)-integers, deferred Nörlund summability, $\varphi_n^{p,q}$-equi-statistical convergence, rate of convergence and Korovkin type approximation theorems.

AMS Subject Classifications: 40A05, 41A36, 40G15

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1 Introduction, definitions and motivation

Let ω be the set of all real valued sequences and suppose any subspace of ω be the sequence space. Let (x_k) be a sequence with real and complex terms. Suppose ℓ_∞ be the class of all bounded linear spaces and let c, c_0 be the respective classes for convergent and null sequences with real and complex terms. We have,

$$\|x\|_\infty = \sup_k |x_k|, \quad (k \in \mathbb{N}),$$

and we recall here that under this norm, the above mentioned spaces are all Banach spaces.

The notion of difference sequence space was initially studied by Kızmaz [27] and then it was extended to the difference sequence of natural order r ($r \in \mathbb{N}_0 =: \{0\} \cup \mathbb{N}$) by defining

$$\begin{aligned}
&\lambda(\Delta^r) = \{x = (x_k) : \Delta^r(x) \in \lambda,\ \lambda \in (\ell_\infty, c_0, c)\},\\
&\Delta^0 x = (x_k); \quad \Delta^r x = (\Delta^{r-1} x_k - \Delta^{r-1} x_{k+1}),\\
&\Delta^r x_k = \sum_{i=0}^{r} (-1)^i \binom{r}{i} x_{k+i},
\end{aligned}$$

(see [19]). Also, these are all Banach spaces under the norm defined by,

$$\|x\|_{\Delta^r} = \sum_{i=1}^{r} |x_i| + \sup_k |\Delta^r x_k|.$$

For more interest in this direction, see the current works [4,7,11–13].

In the investigation of sequence spaces, statistical convergence has got tremendous importance over traditional (classical) convergence in the sense that, here the convergence of a sequence need not required that almost all elements are to fulfil the convergence condition. The convergence condition requires to be executed only for a majority of elements, provided the other elements have natural density zero. The basic idea of statistical convergence was initially studied by Fast [20] and Steinhaus [39]. As of late, statistical convergence has been an active area of research due mainly to the fact that, it is more broad than the usual ordinary convergence and such hypothesis is discussed in the study in the subjects of (for instance) Fourier Analysis, Approximation Theory and Number Theory. For more study in this direction, see the current works [15,17,22,24,26,28,31,33–36].

Let the set of natural numbers be $\mathbb{N}$ and suppose that, $K \subseteq \mathbb{N}$. Also let

$$K_n = \{k : k \leqq n \text{ and } k \in K\}.$$

The asymptotic density of K is given by

$$d(K) = \lim_{n\to\infty} \frac{1}{n} |\{k : k \leqq n \text{ and } k \in K\}|,$$