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Analytic Functions Related to the Strip Domains Involving Generalized Sălăgean Operator

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Abstract. In the paper, the authors introduce a new subclass of univalent functions associated with the strip domains by using the generalized Sălăgean operator. The bounds of coefficients and Fekete-Szegö inequality for functions in this class are obtained. The results presented here extend some of the earlier results.

Key Words: Analytic functions, starlike functions, strip domain, differential subordination, Sălăgean operator.

AMS Subject Classifications: 30C45, 30C80

1 Introduction

Let $\mathcal{A}$ denote the class of functions $f(z)$ normalized by

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{1.1}$$

which are analytic and univalent in the unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$.

Let $f(z)$ and $g(z)$ be analytic functions in $\mathbb{U}$. $f(z)$ is said to be subordinate to $g(z)$, written by

$$f(z) \prec g(z), \quad (z \in \mathbb{U}).$$

If there exists a Schwarz function $\omega(z)$, analytic in $\mathbb{U}$, with

$$\omega(0) = 0 \quad \text{and} \quad |\omega(z)| < 1,$$

such that

$$f(z) = g(\omega(z)), \quad (z \in \mathbb{U}).$$

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It is well known that

$$f(z) \prec g(z), \quad (z \in \mathbb{U}) \implies f(0) = g(0) \quad \text{and} \quad f(\mathbb{U}) \subset g(\mathbb{U}).$$

If $g(z)$ is univalent, then

$$f(z) \prec g(z), \quad (z \in \mathbb{U}) \iff f(0) = g(0) \quad \text{and} \quad f(\mathbb{U}) \subset g(\mathbb{U}).$$

Further, let $\mathcal{P}$ denote the class of functions $p(z)$ of the form

$$p(z) = 1 + \sum_{n=1}^{\infty} c_n z^n, \tag{1.2}$$

which are analytic and convex in $\mathbb{U}$. If $p(z) \in \mathcal{P}$ satisfies the condition

$$\Re(p(z)) > 0, \quad (z \in \mathbb{U}),$$

then, we call the functions the Carathéodory Lemma (e.g., see [2]).

A function $f(z) \in \mathcal{A}$ is said to be starlike of order α and convex of order α in $\mathbb{U}$ if it satisfies

$$\Re\left(\frac{zf'(z)}{f(z)}\right) > \alpha, \qquad (0 \le \alpha < 1;\ z \in \mathbb{U}),$$
$$\Re\left(1 + \frac{zf''(z)}{f'(z)}\right) > \alpha, \qquad (0 \le \alpha < 1;\ z \in \mathbb{U}).$$

This class denotes by $S^*(\alpha)$ and $K(\alpha)$ introduced by Robertson [12]. Let $\mathcal{S}^*(0) = \mathcal{S}^*$ and $\mathcal{K}(0) = \mathcal{K}$, respectively.

Let $\mathcal{M}(\beta)$ and $\mathcal{N}(\beta)$ be the class of functions of $f(z) \in \mathcal{A}$ which satisfy

$$\Re\left(\frac{zf'(z)}{f(z)}\right) < \beta, \qquad (\beta > 1;\ z \in \mathbb{U}),$$
$$\Re\left(1 + \frac{zf''(z)}{f'(z)}\right) < \beta, \qquad (\beta > 1;\ z \in \mathbb{U}).$$

The class $\mathcal{M}(\beta)$ and $\mathcal{N}(\beta)$ are investigated by Uralegaddi et al., respectively [15].

Definition 1.1 ([3]). *Let $-1 \le B < A \le 1$, $C \neq D$ and $-1 \le D \le 1$. The function $p(z) \in P(A,B;C,D)$ if and only if $p(z)$ satisfies the following two subordination relationships*

$$p(z) \prec h_1(z) = \frac{1+Az}{1+Bz}, \tag{1.3a}$$
$$p(z) \prec h_2(z) = \frac{1+Cz}{1+Dz}. \tag{1.3b}$$

For $A = 1-2\alpha$, $(0 \le \alpha < 1)$, $B = -1$, $C = 1-2\beta$, $(\beta > 1)$ and $D = -1$, then

$$p(z) \in P(\alpha,\beta) = P(1-2\alpha,-1;1-2\beta,-1) \iff \alpha < \Re(p(z)) < \beta. \tag{1.4}$$