Commun. Comput. Phys.
doi: 10.4208/cicp.OA-2024-0050

Vol. **37**, No. 1, pp. 171-192
January 2025

Electromagnetic Gyrokinetic Simulation of Tokamak Plasma with Semi-Lagrangian Scheme

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Received 8 March 2024; Accepted (in revised version) 8 September 2024

Abstract. The re-splitting δf method (Ye et. al CPC 2020) has been implemented into the global semi-Lagrangian gyrokinetic code NLT (Lei Ye et al. 2016, JCP) for the simulations of electromagnetic micro-turbulence in tokamak plasmas. The re-splitting method can be incorporated with the numerical Lie transform method, which is an essential numerical scheme for NLT code, to mitigate the cancellation problem appears in the gyrokinetic $p_{\|}$-formulation with gyrokinetic ions and drift-kinetic electrons. With this method, the ion temperature gradient mode (ITG), the kinetic ballooning mode (KBM) and trapped electron mode (TEM) are simulated by NLT and the results are well benchmarked with other gyrokinetic simulation codes.

AMS subject classifications: 65D05, 65Z05, 68U20, 82D10

Key words: Electromagnetic gyrokinetic, numerical Lie transform, re-splitting, semi-Lagrange.

1 Introduction

Large-scale gyrokinetic simulation has become a powerful tool to study various critical physical processes in tokamak plasmas, especially the low frequency ($\omega \ll \omega_c$) waves including but not limited to micro-turbulence and energetic particle driven modes. Here, ω and ω_c are the frequency of relevant waves and gyro-motion of particles. In gyrokinetic theory, the high-frequency gyromotion of charged particles in a magnetic field can be decoupled from the low-frequency drift motion of the gyrocenter, while essential kinetic effects are retained, like finite Larmor radius (FLR) effects and wave-particle resonance effects (through drift motion).

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It is generally accepted that the anomalous transport observed in tokamak experiments are mainly induced by the electrostatic drift-wave type turbulence, such as ion temperature gradient (ITG) mode, trapped electron mode (TEM) and electron temperature gradient (ETG) mode. Therefore, electrostatic gyrokinetic simulations can be applied to study turbulence transport, especially for low-β plasmas, where $\beta = p/(B^2/2\mu_0)$ corresponds to the ratio between plasma kinetic pressure and magnetic pressure. However, a high-β operating scenario is more appealing for economical fusion reactors because the fusion power density is roughly proportional to p^2. As the value of β increases, the magnetic component of perturbations also become increasingly important. The electromagnetic kinetic ballooning modes (KBM) or Alfv*é*nic ion temperature gradient modes (AITG) are likely to be excited due to the coupling between drift-wave and shear Alfv*é*n wave. Moreover, global shear Alfv*é*n eigenmodes, which are essentially electromagnetic waves, can also be driven unstable by the energetic particles through wave-particle resonances. Therefore, comprehensive investigations of these important physical processes call for electromagnetic gyrokinetic simulations. Consequently, development of algorithms for global gyrokinetic simulation, with fully electromagnetic and kinetic electron effects, has been an active area of research due to its significance and challenges in studying tokamak physics.

There are two major challenges confronting electromagnetic gyrokinetic simulation with (gyro/drift)kinetic electrons, which are basically originated from the huge mass ratio between ion and electron. On the one hand, the fast streaming of passing electrons along the field line can impose stringent limitation on the time step size of simulation owing to the Courant-Friedrichs-Lewy (CFL) condition. This restriction can be alleviated either by implicit electron algorithm for continuum codes [3, 10] and PIC codes [19], or by the split-weight scheme [4] or the control-variate method [14] for PIC codes. On the other hand, the well-known 'cancellation problem' [5] arises in the gyrokinetic Ampère's equation with the $p_{\|}$-formulation [13]. The adiabatic current term carried by the zeroth electron distribution function (also referred to the electron skin depth term) can induce numerical accuracy problem in many situations, such as high-β or low-n cases, and should be treated numerically with special care. Here, n is the toroidal mode number. Recently, a novel pull-back mitigation (PBM) scheme has been developed and implemented in the PIC code GYGLES, ORB5 and XGC [7,16,20,23]. Additionally, a re-splitting scheme has also been proposed incorporated with the split-weight scheme and implemented in GEM code [31].

In this work, we report the recent developments for electromagnetic simulation in the semi-Lagrangian gyrokinetic code NLT [34], which is based on the numerical Lie transform method. The re-splitting scheme has been successfully implemented into the NLT code. It has been observed that this algorithm is compatible with the semi-Lagrangian scheme and effectively mitigates the cancellation problem, thereby enhancing the numerical accuracy for electromagnetic turbulence simulations.

The rest of the paper is structured as follows: in Section 2, we present the fundamental gyrokinetic equations, with special attention to the additional field equations resulting