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A Parallel Fully Implicit Unstructured Finite Volume Lattice Boltzmann Method for Incompressible Flows

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Abstract. Lattice Boltzmann method is a popular approach in computational fluid dynamics, and it can be used explicitly or implicitly. The explicit methods require small time step size which is not desirable. In the fully implicit case, existing approaches either lack a scalable and robust parallel nonlinear solver, or don't allow the mesh to be fully unstructured preventing the method to be used for the simulation of fluid flows in large domains with complex geometry. In this paper, a parallel fully implicit second-order finite volume lattice Boltzmann method for incompressible flows on unstructured grids is introduced. The lattice Boltzmann equation is discretized by a finite volume method in space and an implicit backward Euler scheme in time. The resulting large sparse nonlinear system of algebraic equations is solved by a highly parallel Schwarz type domain decomposition preconditioned Newton-Krylov algorithm. The proposed method is validated by three benchmark problems with a wide range of Reynolds number: (a) pressure driven Poiseuille flow, (b) lid-driven cavity flows, and (c) viscous flows passing a circular cylinder. The numerical results show that the proposed method is robust for all the test cases and a superlinear speedup is obtained for solving a problem with over ten million degree of freedoms using thousands of processor cores.

AMS subject classifications: 76M12, 76M28, 76D99, 76F99
Key words: Lattice Boltzmann equation, fully implicit method, unstructured grids, Newton-Krylov-Schwarz, parallel computing.

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1 Introduction

In the past 30 years, the standard lattice Boltzmann method (SLBM) has become an effective and promising approach in computational fluid dynamics. In SLBM, the lattice

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Boltzmann equation (LBE) is solved to simulate the flow on uniform Cartesian grids with different collision models [1–4] in which the grid spacing is equal to the time step, and the spatial grid has to be a uniform Cartesian gird in order to execute the stream operation along the discrete velocity set. The Courant-Friedrichs-Lewy (CFL) number for SLBM has to be 1. To simulate incompressible flow problems defined on geometrically complex domains or flows with strong local gradients, SLBM requires a uniformly fine grid in order to capture the details, but an unstructured grid method only requires a finer grid near the body or the walls, and the grid for the far field can be quite coarse. This results in a much smaller overall grid. Moreover in SLBM, the staircase patterns on some of the boundaries may introduce geometrical irregularities, even on smooth surface due to the restrictions of the uniform Cartesian grid, and the fitting accuracy on the curved boundary is lower than that of a body-fitted grid. In recent years, a few off-lattice Boltzmann methods were devised to avoid the Cartesian grid restriction including grid refinement and multi-block methods [5, 6], hybrid lattice Boltzmann method [7, 8], finite difference lattice Boltzmann methods (FD-LBMs) [9–11], continuous or discontinuous finite element lattice Boltzmann methods [12, 13], finite volume lattice Boltzmann methods (FV-LBMs), the lattice Boltzmann flux solver [14] and the discrete unified gas kinetic scheme in the continuum limits [15].

In this paper, we introduce and study a fully implicit and fully unstructured version of FV-LBM originally proposed by Nannelli and Succi [16, 17], in which a coarse-grained distribution is defined via a cell averaging operator over a control volume, and a first-order Euler time-marching scheme is employed for the time integration. Recently, FV-LBM has been extended to unstructured grids. Patil et al. presented a cell-centered FV-LBM with total-variation diminishing formulation on 2D unstructured grids and they provided the Chapman-Enskog (CE) analysis for their FV-LBM with a forward Euler time integration on a special grid comprised of equilateral triangles [18, 19]. Misztal et al. presented a FV-LBM with a vertex-centered scheme, and studied two temporal discretizations (forward Euler method and the operator splitting method) on 2D unstructured grids yielding the Navier-Stokes equations through the CE expansion [20]. Li and Luo proposed an explicit cell-centered FV-LBM with a multiple-relaxation time collision model on arbitrary unstructured grids [21], in which the fluxes on the cell interfaces are evaluated by a low-diffusion Roe scheme. Subsequently, they presented a gas kinetic BGK scheme to evaluate the fluxes based on the formal analytical solution of the lattice Boltzmann BGK equation [22]. Chen and Schaefer devised a simple unified Godunov-type upwind approach which does not need Riemann solvers for the face flux calculation on 2D unstructured cell-centered triangular grids [23]. In our previous work, we developed a scalable parallel explicit FV-LBM for 3D incompressible flows [24], and extended the method to thermal incompressible flows [25] and inviscid high-speed compressible flows [26].

Note that all the works mentioned above are explicit or semi-implicit schemes whose time step size is constrained by the CFL number. To enhance the stability with larger time step sizes, a few fully implicit LBMs were proposed. Cevik and Albayrak proposed