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Three Dimensional Dynamics of Polar Fluids: A Lattice Boltzmann Approach

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Abstract. In this paper we propose a Lattice-Boltzmann-based mesoscopic model for the three-dimensional flow of an incompressible polar fluid.

The mesoscopic model is equivalent to the usual incompressible three-dimensional Navier-Stokes equation coupled with the angular momentum equation, which describes the evolution of the angular velocity vector of the fluid particles.

The proposed model is applied to investigate the effects of the fluid polar structure on three steady flows: the steady Couette and Poiseuille flows in a square channel and the three-dimensional lid-driven cavity flow at $Re=100, Re=400$.

The effects of the fluid polar structure on the above mentioned flows are investigated by varying the relevant dimensionless parameters: the coupling parameter N and the geometric parameter L.

Results are consistent with the predictions of the theory of polar fluids. In particular, it is shown for the three-dimensional lid-driven cavity flow that the effect of the coupling parameter N is to lower the effective Reynolds number and thus to increase the viscosity.

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Key words: Computational fluid dynamics, lattice Boltzmann method, polar fluids.

1 Introduction

Polar fluids are fluids whose microstructure is mechanically significant [5]. Interactions between fluid particles of a polar fluid include forces as well as torques. Under the action of the latter the fluid particles develop an angular velocity field which has to be described

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by a suitable kinematic variable [5]. Moreover a non vanishing skew-symmetric part of the stress is assumed to balance the action of the distributed torque [5]. Thus constitutive equations for the skew symmetric stress tensor and a vector equation for the angular velocity field have to be introduced in the set of governing equations.

The study of polar fluids is of great importance in many scientific and industrial applications of complex fluids, such as flows of exotic lubricants [18], liquid crystals [16], dense suspensions of solid particles [2], granular materials [24], blood [13], colloidal suspensions of magnetic particles in a non-magnetic liquid host [9]. Moreover, the behaviour of the so-called active matter, i.e. materials composed of individual units that consume energy to generate motion or mechanical work, can be modeled as that of a polar fluid. Therefore the study of the dynamics of polar fluids presents many aspects that are still open and worthy of scientific investigation.

Two-dimensional flows of polar fluids have been considered recently: [3, 15, 20, 25]. In the two-dimensional case the angular velocity component normal to the plane of motion, the only non vanishing component of the angular velocity, is determined by an advection-diffusion equation with a source term depending on the velocity field. The velocity field is determined by the mass conservation and Navier-Stokes equations, with a forcing term dependent on the angular velocity, which can then be considered as an active scalar quantity.

Three-dimensional flows of polar fluids pose noteworthy additional complexity due to the need to account for all three components of the angular velocity vector field: the velocity field transports and diffuses throughout the fluid domain the vector field of the angular velocity, which in turn exerts a forcing feedback action on the velocity field. The main complexity is due to the fact that the angular velocity field is fully three-dimensional and requires three scalar equations, one for each component of the rotation velocity vector field [22].

The numerical approach is the only viable to address such a complex mathematical setting. In the literature there are examples of computational methods applied to the simulation of fully three-dimensional polar fluid flows [19, 22] based on consolidated algorithms while, to our best knowledge, mesoscopic approaches based on kinetic theory have so far only been proposed for $2D$ scenarios [15].

There are good reasons to favor the mesoscopic approach based on the Lattice Boltzmann method: indeed it has proven to be a particularly successful choice for tackling complex multiphysics problems thanks to its roots in kinetic theory, algorithmic simplicity, and intrinsic amenability to parallelization [23].

Thus the aim of this paper is to extend the Lattice Boltzmann-based computational method presented in [15] in order to simulate fully three dimensional flows of polar fluids.

The structure of the paper is as follows: the governing equations of the polar fluid and the mesoscopic Lattice Boltzmann-based formulation are briefly illustrated in Section 2. Relevant dimensionless parameters and case studies are considered in Section 3. The latter are the steady three dimensional Couette and Poiseuille flow and the cubic lid-drive