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An Efficient Reduced-Order Model Based on Dynamic Mode Decomposition for Parameterized Spatial High-Dimensional PDEs

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Abstract. Dynamic mode decomposition (DMD), as a data-driven method, has been frequently used to construct reduced-order models (ROMs) due to its good performance in time extrapolation. However, existing DMD-based ROMs suffer from high storage and computational costs for high-dimensional problems. To mitigate this problem, we develop a new DMD-based ROM, i.e., TDMD-GPR, by combining tensor train decomposition (TTD) and Gaussian process regression (GPR), where TTD is used to decompose the high-dimensional tensor into multiple factors, including parameter-dependent and time-dependent factors. Parameter-dependent factor is fed into GPR to build the map between parameter value and factor vector. For any parameter value, multiplying the corresponding parameter-dependent factor vector and the time-dependent factor matrix, the result describes the temporal behavior of the spatial basis for this parameter value and is then used to train the DMD model. In addition, incremental singular value decomposition is adopted to acquire a collection of important instants, which can further reduce the computational and storage costs of TDMD-GPR. The comparison TDMD and standard DMD in terms of computational and storage complexities shows that TDMD is more advantageous. The performance of the TDMD and TDMD-GPR is assessed through several cases, and the numerical results confirm the effectiveness of them.

AMS subject classifications: 35K05, 76D05

Key words: Parameterized time-dependent PDEs, tensor train decomposition, dynamic mode decomposition, incremental singular value decomposition.

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1 Introduction

A reduced-order model (ROM) is constructed as the surrogate model of the high-fidelity (HF) simulation model, which has become a popular methodology to reduce the computational complexity of numerical problems with controllable prediction errors and has received a lot of attention in practical applications [4, 9, 18, 23, 33, 42]. ROM typically assumes that there is a low-dimensional subspace in which the solution manifold can be approximated well and then constructs a low-dimensional representation of the HF solution on the subspace. Dynamic mode decomposition (DMD) is a common technique in the construction of ROM, which was proposed firstly by Schmid and Sesterhenn [30] and has attracted a lot of attention from researchers [37,39,41]. DMD can approximate the state of the dynamical system at any time instant according to the coherent spatio-temporal structures extracted from HF snapshots. DMD-based ROMs are originally developed for the time-dependent PDEs [7, 20, 21, 27, 32], and the comparison results in [10, 22] showed that DMD-based ROMs outperform those based on proper orthogonal decomposition (POD) in terms of the extrapolation in time. However, despite these advantages, the characteristics of DMD make it impossible to be applied directly to parameterized PDEs. That is, the DMD modes and eigenvalues used to derive the approximate solutions vary with the parameter values. Recently, some efforts have been made to apply DMD to parameterized PDEs, e.g., Ref. [14] carried out the parameterized DMD by interpolating the DMD eigenpair and DMD operator; In [1], all the snapshots were projected to the reduced space and then DMD models were constructed based on the projection coefficients corresponding to each training parameter value, which were employed to approximate the reduced snapshots for future instants, and a regression technique was used to implement the parameterized part of the ROM.

Although these DMD-based ROMs are applied to high-dimensional spatial equations, the solutions of PDEs need to be straightened spatially, which not only makes the model less interpretable, but also encounters high computational and storage costs. For example, for the spatial 3-dimensional PDEs, assume that the number of spatial discretization elements are n_1,n_2 and n_3 respectively, then the solution for any given parameter and time values is a three-dimensional matrix. The required storage cost for each straightened snapshot is $\mathcal{O}(n^3)$, where $n = \max\{n_1,n_2,n_3\}$. That is, suppose there are n_p parameter values and the snapshots at n_t time instants for each parameter value are used to construct the DMD-based ROMs, the storage cost for all the snapshots is $\mathcal{O}(n^3 n_t n_p)$. Furthermore, when SVD is performed to a set of such snapshots, the size of the resulting spatial basis is $n^3 \times r$, where r is the number of basis, which will cause the computational costs of the subsequent construction of the DMD approximations become significant as n grows. The low-rank approximation can mitigate these issues without straightening the snapshot matrix and some related approaches have been proposed. Dynamical low-rank approximation method [16] is developed for the approximation of time-dependent data matrices. A dynamically orthogonal (DO) approximation is introduced for time-dependent stochastic PDEs in Ref. [29]. The properties of the DO approximation and