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The p-Weighted Limiter for Nodal Discontinuous Galerkin Methods on Two Dimensional Mixed Grids

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Abstract. Mixed grids offer advantages in grid generation, calculation efficiency and numerical accuracy. The p-weighted limiter works effectively on 1D, triangular and tetrahedral grids for the discontinuous Galerkin (DG) method. Whereas the generalization to unstructured mixed grids for nodal DG methods is not straightforward. On quadrilateral grids, the tensor-product of solution points for nodal DG methods makes it complex and time-consuming to construct bi-linear candidate polynomials and perform weighted summation with the tensor-product polynomials. A simple yet effective way is proposed to address this issue. The linear candidate polynomial is constructed by interpolating values on the face and performing a forward difference in the central troubled cell. The truncation strategy on the linear weights is also improved. The weighted summation is carried out dimension-by-dimension on local coordinates as the 1D case. Since the limiting only depends on the central cell type, the final implementation for nodal DG methods on mixed grids is straightforward. A parameter-free shock detector is proposed to identify the troubled cells near shock waves. Several inviscid, laminar and turbulent flows with shocks are employed to showcase the efficiency of the improved p-weighted limiter and shock detector on unstructured mixed grids for nodal DG schemes.

AMS subject classifications: 35L67, 65M60

Key words: Discontinuous Galerkin, shock capturing, unstructured mixed grids, p-weighted limiter, compressible turbulent flows.

1 Introduction

The use of unstructured mixed grids is essential for achieving higher efficiency in both mesh generation and calculation. This is particularly important for viscous turbulent

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flows, where grids with large aspect ratios are required to accurately capture anisotropic flows such as boundary layers and stretched vortices. In terms of discretization schemes, high-order methods are widely employed to capture flow details effectively. Among these methods, discontinuous Galerkin (DG) methods [4] has emerged as one of the most popular approaches due to its good mathematical properties for stability and ability to work on arbitrary grids. When it comes to efficiency, the methods such as DGSEM [6, 9] and FR [16,37] on tensor-product grids with tensor product solution points offer superior performance than other types of elements.

One challenge faced by the high-order methods is the shock capturing strategy especially on mixed grids. On mixed grids, modal type DG methods can use uniform discontinuities capturing way [14]. The artificial viscosity and filtering methods usually provide identical way to suppress numerical oscillations on different kinds of grids [35, 44]. However, nodal type DG methods require different limiting procedures that take into account the specific nodal arrangements. The widely studied limiter such as Hermite WENO [29] and moment [19] limiters are often applied on the modal type DG methods. However, their application to nodal-type DG methods on mixed grids presents certain challenges. The multi-resolution WENO limiter have been proposed for the high-order finite difference and finite volume schemes [46], and subsequently generalized to the DG methods in solving inviscid and viscous flows on triangular or quadrilateral grids [27,45]. The MOOD limiter has been applied individually on unstructured triangular/tetrahedral grids and tensor-product grids [7, 8]. One more challenge is that grids with large aspect ratio are always employed for the viscous flow especially turbulent flows in the boundary layer and stretched vortex regions. The shock capturing ability and the limiter's accuracy would be affected by the geometrical factor. Regions with large flow gradients often demand high-accuracy numerical schemes to accurately capture detailed flow features.

The shock detector is usually employed to reduce the computational cost associated with the high-order shock capturing ways and to preserve the high-order accuracy in smooth regions for the low-order shock capturing method. A widely used detector would be the KXRCF detector [20]. One issue of this detector is that it would mistake the region with large gradient as troubled cells such as the region near stagnation points of aerofoil. Reference [25] shows that the KXRCF would mistake the smooth cells as troubled ones especially for higher order schemes and some modifications are made on the judgment and threshold [24]. The misjudgment becomes more severe inside the boundary layer due to large flow gradient and large grid aspect ratio. The discontinuity sensor originated from Persson's approach [33] has also been adopted in different artificial viscosity methods [2, 31] to reflect the shock region.

This paper focuses on the development of a straightforward yet effective limiter for nodal DG schemes in solving inviscid and viscous flow on 2D mixed grids. The nodal arrangement of DG methods consists of Gauss-Lobatto type points in triangles [13] and the tensor-product of Gauss-Legendre (GL) quadrature points in quadrilaterals. The p-weighted limiter proposed in [23, 26] is applied to triangular grids, while a dimension-by-dimension limiting approach with a new linear candidate reconstruction method from