Commun. Comput. Phys.
doi: 10.4208/cicp.OA-2023-0259

Vol. **36**, No. 5, pp. 1339-1377
November 2024

An Improved Modified Ghost Fluid Method for Compressible Multi-Medium Fluid Flows

Xiaotao Zhang[1], Chengliang Feng[1], Changsheng Yu[2], Yibo Wang[1] and Tiegang Liu[1,*]

[1] *LMIB and School of Mathematical Sciences, Beihang University, 37 Xueyuan Road, Haidian District, 100191, Beijing, P.R. China.*
[2] *Institute of Mathematics, Johannes Gutenberg-University Mainz Staudingerweg 9, 55 128 Mainz, Germany.*

Communicated by Michael Dumbser

Received 2 October 2023; Accepted (in revised version) 20 April 2024

Abstract. Pressure dislocation might be observed when the modified ghost fluid method (MGFM) is applied to simulate radially symmetric multi-medium fluid flows for a long time. We disclose the insightful reason that the MGFM cannot satisfy balanced boundary conditions when applied to simulate the radially symmetric compressible multi-medium flows, resulting in a first-order temporal error in the interface region. To impose these balanced boundary conditions, we develop an improved MGFM in this work. The reconstruction for the initial value of the multi-medium Riemann problem at the interface is specially designed in combination with these balanced boundary conditions. In addition, the predicted instantaneous interface states and balanced boundary conditions are then utilized to define the ghost fluid states. Theoretical analysis shows that the improved MGFM can satisfy these balanced boundary conditions and effectively eliminate the first-order temporal error at the interface. Its extension to two dimensions is also presented. Numerical results show that the proposed improved MGFM can restraint pressure dislocation and overheating at the material interface very well and effectively improve mass conservation.

AMS subject classifications: 65M22, 65M60

Key words: Multi-medium compressible flows, radially symmetric flows, modified ghost fluid method, balanced boundary conditions.

*Corresponding author. *Email addresses:* liutg@buaa.edu.cn (T. Liu), xiaotaozhang@buaa.edu.cn (X. Zhang), charlon_feng@buaa.edu.cn (C. Feng), ChangshengYu@buaa.edu.cn (C. Yu), yibowang@buaa.edu.cn (Y. Wang)

1 Introduction

In the numerical simulation of multi-medium flows, the ghost fluid methods (GFM) have been common approaches for treating the immiscible interfaces [1–7]. Among them, a typical representative is the modified ghost fluid method (MGFM) [5], which establishes and reassigns the ghost fluid values by solving a multi-medium Riemann problem at interfaces and has been successfully applied to solve various problems involving large density ratios, large pressure ratios [5,6,8–19]. Based on the techniques of multi-medium Riemann problem, the interface interaction GFM (IGFM) [6], the real GFM (RGFM) [7], the practical GFM (PGFM) [8], the second-order MGFM (2nd-MGFM) and so on, have been developed.

The MGFMs predict the moving interface states based on the classical Riemann problem with piecewise constant initial values, theoretically resulting in only first-order accuracy of the interface treatment. When they are employed for long-time simulation of multi-medium problems with geometrical source terms, there is always a first-order temporal error occurring at the interface. Accumulation of such errors with time leads to numerical overheating and pressure dislocation [20]. To eliminate the error, Liu and co-workers [20–22] and Li and co-workers [23] developed high-order sharp interface methods based on the generalized Riemann problem (GRP) [24–27], and achieved to control the conservation error and eliminate the pressure dislocation. Nevertheless, implementing the above two methods relies on predicting interface states and spatial derivatives solved from the complex multi-medium GRP. At present, an explicit form for the first-order derivatives in GRP is not available for a general equation of state.

To avoid solving a complex multi-medium GRP, we develop an improved MGFM, especially for compressible multi-medium fluid flows, by maintaining the velocity and pressure equilibrium through enforcing the interface balanced boundary conditions in this work. The construction for the multi-medium Riemann problem and the assignment of ghost fluid states are modified to satisfy the balanced boundary conditions. It can be shown that the proposed method theoretically satisfies the balanced boundary conditions. This new approach can be regarded as a direct extension of MGFM when applied to the radially symmetric problem to maintain the velocity and pressure balance across the interface. The presented method possesses the following appealing features: simplicity and efficiency. A series of typical test cases for 1D planar, cylindrical, and spherical symmetric and 2D planar problems are provided to verify the maintenance of balanced boundary conditions and improvement of mass conservation for the proposed method.

This article is organized as follows. Section 2 introduces governing equations and the error analysis of MGFM for radially symmetric multi-medium problems. In Section 3, we propose an improved MGFM and analyze the error at the interface. In Section 4, we apply the proposed method to solve various compressible multi-medium fluid flows. The conclusion is briefly given in Section 5.