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A Lightweight Data-Driven Model Enhanced Discontinuous Galerkin Method for Rapidly Simulating Transonic Airfoil Flowfields

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Abstract. Accurate and rapid prediction of flowfields is crucial for aerodynamic design. This work proposes a discontinuous Galerkin method (DGM) whose performance can be enhanced with increasing data, for rapid simulation of transonic flow around airfoils under various flow conditions. A lightweight and easily updatable data-driven model is built to predict roughly correct flowfield, and the DGM is then utilized as the CFD solver to refine the detailed flow structures and provide the corrected data. During the construction of the data-driven model, a zonal proper orthogonal decomposition (POD) method is designed to reduce the dimensionality of flowfield while preserving more near-wall flow features, and a weighted distance-based radial basis function (RBF) is constructed to enhance the generalization capability of flowfield prediction. Numerical results demonstrate that the lightweight data-driven model can predict the flowfield around a wide range of airfoils at Mach numbers ranging from 0.7 to 0.95 and angles of attack from -5° to 5° by learning from sparse data, and maintains high accuracy of the location and essential features of flow structures (such as shock waves). In addition, the data-driven model enhanced DGM is able to improve the computational efficiency and simulation robustness as compared to normal DGMs in simulating transonic inviscid/viscous airfoil flowfields on arbitrary grids, and further enables rapid aerodynamic evaluation of numerous sample points during the surrogate-based aerodynamic optimization.

AMS subject classifications: 65M60, 68Q32

Key words: Data-driven model, discontinuous Galerkin methods, CFD simulation of transonic flowfield, aerodynamic optimization.

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1 Introduction

Modern industrial aerodynamic design demands heightened design efficiency and exceptional aerodynamic performance of the final designs. In the conceptual design stage, surrogate-based optimization (SBO) [16,27,34,39] can achieve more comprehensive and superior aerodynamic performance in optimization results due to its ability to better explore the design space. However, when addressing high-dimensional design space, SBO is recognized expensive due to the requirement of assessing quickly increasing sample or design points for sufficient coverage of the design space [36].

The primary expense within SBO for evaluating aerodynamic performance at specific design points primarily comes from using computational fluid dynamics (CFD) to simulate the flowfields around continuously changing aerodynamic shapes under various flow conditions. Hence, the CFD solver needs to be robust, efficient but still accurate. Our previous works [13,14] have demonstrated that high-order DGMs [15,19] are capable of providing highly reliable aerodynamic quantities and sensitivities even on coarse grids. However, DGMs still need improvements in terms of robustness and computational cost when treating intricate especially 3D shapes or large-scale computational grids.

Over the past few decades, the reduced-order models (ROMs) [2], particularly non-intrusive reduced-order models (NIROMs) [10], provide attractive alternatives to alleviate the high computational cost of CFD simulation. The key idea of the NIROMs for flowfield prediction is to construct a map between the design variable and the reduced-dimensional flowfield based on CFD simulation data. Classical dimensionality reduction methods include POD, dynamic mode decomposition (DMD), and manifold learning [17,28], etc. POD [3] and DMD [1] utilize a low-dimensional linear space spanned by a set of basis to approximate the high-dimensional space, and are widely used in fluid dynamics due to their simplicity and interpretability. After dimensionality reduction, appropriate interpolation or fitting model, such as RBF, Kriging interpolation, Gaussian regression, is constructed with an acceptable loss of accuracy to replace the original full-order model (CFD). NIROMs that combine POD/DMD with interpolation are easily implementable, conveniently updatable, and low computational cost. However, linear dimensionality reduction methods might lose intricate nonlinear characteristics of data [5]. Additionally, for high-dimensional design variables encompassing both geometric and flow state parameters, classical interpolation mappings sometimes exhibit limited predictive performance beyond the interpolated dataset.

Recently, deep neural networks (DNNs) have been developed to accomplish the task of flowfield prediction. The DNN directly establishes a map between high-dimensional geometry or grid and full-order flowfield through its exceptional nonlinear encoding and decoding capabilities. Typical DNNs used to predict flowfield include convolutional neural networks (CNNs) [11], U-Nets [4], generative adversarial networks (GANs) [18], attention-based nets [9,41], PointNets [25], graph convolutional networks (GCNs) [32], etc. CNNs are widely used in the field of computer vision for feature extraction, and exhibit a remarkable capacity to reduce the number of training parameters as compared