Commun. Comput. Phys.
doi: 10.4208/cicp.OA-2024-0135

Vol. **38**, No. 5, pp. 1417-1452
November 2025

The Multidomain Hybrid Discontinuous Galerkin Method and Finite Difference Method for Solving Compressible Navier-Stokes Equations on Hybrid Meshes

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Communicated by Michael Dumbser

Received 13 June 2024; Accepted (in revised version) 22 October 2024

Abstract. In this work, we develop a multidomain hybrid discontinuous Galerkin (DG) method and finite difference (FD) method for solving two-dimensional compressible Navier-Stokes equations on the hybrid meshes. The direct discontinuous Galerkin (DDG) method and central difference (CD) scheme are utilized to discretize the viscous fluxes respectively. This approach combines the flexibility for the complex geometries of the DG method on the unstructured meshes, and the computational efficiency of the FD method on Cartesian grids. At the artificial interfaces between the DG subdomain and FD subdomain, the square ghost cells are generated and the weighted essentially non-oscillatory (WENO) interpolation is employed to reconstruct the degrees of freedom of these ghost cells. To ensure the accuracy in smooth regions and the correct position of the shock wave, the troubled cell indicator is adopted to determine the nonconservative or conservative coupling modes. The construction process of the numerical fluxes at the artificial interfaces is described specifically and the corresponding WENO interpolation coefficients are given in detail. Numerous numerical results demonstrate that the multidomain hybrid method achieves high-order accuracy in smooth regions, robustness in shock simulations, flexibility in handling complex geometries, and significant computational cost savings compared to the traditional DG method on the hybrid meshes.

AMS subject classifications: 65M60, 65M06, 65M99, 35L65, 35K20

Key words: Discontinuous Galerkin method, finite difference method, multidomain hybrid method, Navier-Stokes equations, hybrid meshes.

1 Introduction

High-order and high-resolution methods have attracted more and more interest recently due to the increased demand to accurately predict engineering problems and understand fundamental flow physics [19,44,45]. For example, the direct numerical simulation (DNS) of turbulence [23, 33] requires high-order discretization methods with low diffusion to resolve different scale turbulence eddies.

Among the various high-order methods, the FD method and the DG method are particularly popular among researchers for different reasons. The FD method [24, 26, 34, 43, 55] is known for its simplicity, computational efficiency and ease of achieving high-order accuracy. It is a grid-based method and straightforward to implement, which is especially attractive for problems on structured grids. On the other hand, the DG method [9–13, 15, 46] offers several advantages, particularly in handling complex geometries, parallel computation, and demonstrating excellent convergence properties, which allows for greater flexibility in handling complex geometries and dealing with irregular boundaries. Indeed, despite their individual strengths, both the FD method and the DG method have their limitations and drawbacks. Recognizing this, researchers have proposed hybrid methods that aim to combine the advantages of different numerical methods.

Fernando et al. [20] proposed a FD scheme based on the DG method, where the FD scheme is derived from the specific form of DG the method, and then applied to the wave propagation. Luo et al. [28] presented a reconstruction-based discontinuous Galerkin (rDG(P1P2)) method, where the quadratic polynomial solution (P2) is obtained from the underlying linear polynomial (P1) discontinuous Galerkin solution using a least-squares method. Zhu and Guo et al. [22, 53] developed a hybrid WCNS-CPR scheme for the efficient supersonic simulations, where WCNS is adopted to capture shocks while the smooth area is calculated by CPR. Tavelli et al. [39] proposed a new arbitrary high-order accurate semi-implicit space-time DG method for the compressible Euler and Navier-Stokes equations on staggered unstructured curved meshes. This method was later extended to the hybrid finite volume/finite element scheme by Dumbser et al. [3]. More recently, Maltsev et al. [32] developed a hybrid DG/FV schemes, in which the key ingredient is a switch between DG method and FV method based on the CWENOZ scheme, and then they extended the work to multi-species problems involving gas-gas and gas-liquid systems [31]. Based on the computational domain decomposition, Cheng et al. [5–7] proposed the multidomain hybrid RKDG and WENO methods for solving the hyperbolic conservation laws, which combined the advantages of high efficiency of the WENO schemes and easy treatment of the complex geometries easily from the DG methods. Later, Zhang et al. [50] analyzed the linear stabilities of the conservative multidomain hybrid methods and introduced two ways of healing the stable problems. Moreover, Wang et al. [42] proposed a novel high-order FD scheme based on DG boundary treatment and no more than two layers were needed for the complex boundary treatments.

In our previous work [49], a class of multidomain hybrid method of direct discontinu-