

An Improved Ghost-Point Technique for the Electrostatic Equation with Curved Boundaries and Discontinuous Permittivity in ECR Ion Source PIC Simulations

Armando Coco^{1,*} and Lorenzo Neri²

¹ *University of Catania, Department of Mathematics and Computer Sciences, Viale Andrea Doria 6, 95125 Catania, Italy.*

² *Istituto Nazionale di Fisica Nucleare – Laboratori Nazionali del Sud, Via Santa Sofia 62, 95125 Catania, Italy.*

Received 16 April 2025; Accepted (in revised version) 18 November 2025

Abstract. In this paper, we propose an efficient Poisson solver for a two-dimensional axisymmetric domain to simulate plasma in a 2.45 GHz Electron Cyclotron Resonance (ECR) ion source. The solver employs a uniform Cartesian mesh for a curved domain, utilizing an unfitted boundary method. The presence of two Boron Nitride disks inside the plasma chamber creates a discontinuity in permittivity, leading to a Poisson problem with discontinuous coefficients. We discretize the Laplace operator by standard central difference formulas, and apply a ghost-point technique at nodes outside the domain and near the interfaces between the disks and the chamber, enforcing appropriate boundary and interface conditions for the ghost values. This method significantly simplifies and accelerates computations compared to boundary-conforming mesh approaches. The primary objective of this work is to upgrade our Poisson solver (employed in the Particle-In-Cell (PIC) code) from a fitted boundary method, such as COMSOL, to a custom implementation based on an unfitted boundary approach. The performance improvements are substantial, reducing computation time from a minimum of 5 seconds per solution with COMSOL to just 0.062 seconds with our custom solver.

AMS subject classifications: 65N06, 65N12, 65N22, 35J05, 78M20, 82D10, 78A30, 35R05

Key words: Ghost-point method, elliptic interface problems, numerical treatment of material discontinuities, finite-difference discretization, embedded boundary techniques, charge-particle simulation, plasma source modeling, computational electromagnetics.

*Corresponding author. *Email addresses:* armando.coco@unict.it (A. Coco), neri@lns.infn.it (L. Neri)

1 Introduction

The Particle-In-Cell (PIC) method [5, 10, 12, 23, 41] is a powerful simulation strategy to compute the dynamics of interacting particles. Plasma study is one of the most representative application cases of PIC. Large numbers of physical particles are represented by a smaller number of simulated macroparticles which move through a computational domain. A mesh discretizes the domain and identifies the structure used to collect particle properties like charge, densities, temperature distributions, etc. Furthermore, the electrostatic, magnetostatic, and electromagnetic external fields applied to the particle motion are stored in the same structure.

At the Istituto Nazionale di Fisica Nucleare – Laboratori Nazionali del Sud (INFN–LNS), the development of a plasma simulation tool began in 2010 [34]. Initially, the computational resources and code efficiency were limited, so we limited the goal to visualize only specific phenomena. Due to challenges in efficiently computing the electrostatic field, a strategy called Stationary-PIC was developed [27, 33], where the electrostatic field is computed only intermittently. Despite the initial limitations, this approach yielded significant results, earning two Geller prizes in 2012 and 2016 [1].

Recently, we have made substantial improvements in our ability to solve the Poisson problem, marking a significant advancement towards predictive software. This paper presents a new strategy we developed for the efficient computation of the electric field in plasma for a two-dimensional axis-symmetric simulation. This enhancement is crucial to expanding the current code’s capabilities, reproducing and fully understanding why the 2.45GHz ECR ion sources HMDIS [31, 32] magnetic configuration can produce an extremely stable beam with easy intensity modulation capability and low emittance. We point out that this work focuses exclusively on the accurate and efficient solution of the electrostatic equation; time integration and particle advancement are beyond the scope of the present study.

In 2011, our initial approach to solving the Poisson problem involved the COMSOL *LiveLink for Matlab* module, which facilitates communication between COMSOL [2] and MATLAB [3]. In this setup, particle motion is computed in MATLAB, and the charge distribution is collected using a square mesh. The charge distribution is stored in an array and written to a text file. Within MATLAB, a COMSOL server is initialized, and the electrostatic model is loaded. The COMSOL model employs a MATLAB function to read the charge density text file and provides the charge distribution for the tetrahedral mesh of the finite-element solver. Due to differences between the COMSOL and MATLAB meshes, a linear interpolation step is included. The main MATLAB program then instructs the COMSOL server to solve the electrostatic model and subsequently requests the electrostatic field data corresponding to the MATLAB mesh, necessitating another interpolation from the COMSOL mesh. This strategy, involving file writing and dual interpolation steps, resulted in poor performance and low accuracy. The overall workflow of data exchange is discussed in the numerical test of Section 4.3.1 and summarized in Fig. 11.

In this paper, we present a recent approach in which the electrostatic field is computed